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The String Scaling of the Upright Piano by John Clemm(?) in its Historical Context

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John Watson's article in this issue of *JAMIS* provides a quite plausible metrological interpretation of the lengths of the *C* strings in the upper half of the compass of the early upright piano in the collection of the Moravian Historical Society (MHS).[1] In light of the deeply rooted historical relationship between organ building and stringed-keyboard instrument making,[2] the article further perceptively suggests that the instrument's unusual non-pythagorean scaling likely resulted from the application of methods derived from the art of scaling organ pipes. The purpose of the present essay is to explore in greater depth the historical context of how this might have been done by the instrument's maker, credibly suggested to have been John Clemm.

Definitions

In stringed instruments, pythagorean (sometimes called "just") scalings are those in which string lengths follow the 2:1 octave ratio. I propose that "Pythagorean" with the upper-case initial be used exclusively for scalings and tunings generated by the traditional Pythagorean ratios for pure intervals, 2:1 for the octave, 3:2 for the fifth, 4:3 for the fourth, and

1. I am most grateful to John Watson for sharing drafts of his article, which led to this, my own pendant effort; for his encouragement, help, and suggestions along the way; also for drawing figs. 15 and 16 from my crude sketches and for making new drawings for use as figs. 7 and 8 before I obtained suitable photos of the originals. I also thank Laurence Libin for various help and suggestions, David Blum of the Moravian Music Foundation for expeditiously providing a photo of the Tannenberg clavichord drawing (fig. 1), and Holger Horstmann of the Stadtarchiv Hannover for likewise providing the photos of Christian Vater's scaling charts. I remain extremely grateful to Alec Cobbe for allowing me, many years ago, to examine his Andreas Ruckers harpsichord, the basis of fig. 14.

2. In addition to my "Some Remarks on the Relationship between Organ and Stringed-Keyboard Instrument Making," *Early Keyboard Journal* 18 (2000): 95–137, see Eva Helenius-Öberg, "Connections between Organ Building and Keyboard Instrument Building in Sweden before 1820," in Sverker Jullander, ed., *GOArt Research Reports* 1 (Göteborg: Göteborg Organ Art Center, 1999), 127–71.

their derivatives, such as 9:8 for the whole tone. I further propose that "pythagorean" with the lower-case initial be used in a more general sense, for scalings with the 2:1 octave ratio but with other intervals determined by ratios other than the strictly Pythagorean (e.g., 5:4 for the pure major third or 24:23 as a close approximation for the chromatic semitone in quarter-comma meantone temperament), or by irrational factors (e.g., the square root of 5/4 as the factor for a whole tone in quarter-comma meantone, or the twelfth root of 2 as the factor for an equally tempered semitone). The use of lower-case "pythagorean" in this general sense continues to acknowledge Pythagoras's legendary discovery of the mathematical relationship between pitch and the lengths of strings or pipes. I would avoid referring to Pythagorean or pythagorean scalings as "just," because this term invites confusion with just intonation, that is, tunings with pure intervals derived from the harmonic series.

Scalings like that of the MHS piano are called "non-pythagorean" or "tapered" in the modern literature. "Tapered" refers to the gradual reduction in the relative lengths of the strings, i.e., their c^2-equivalent lengths,[3] for each successive note below the highest.[4] "Tapered scale" seems first to have been used by William R. Thomas and J.J.K. Rhodes in 1967 as the equivalent of Frank Hubbard's term "foreshortening," meaning the relative shortening of strings in the lower half of an instrument's compass.[5] Later writers, such as Grant O'Brien, have used the term in its current sense of scalings in which the tapering begins at the top of the compass.[6] I propose that "tapered"/"tapering" be reserved for this sense, and

3. A string's c^2-equivalent length is its actual length times the pythagorean factor between its note and c^2. For example, if a c^1 string is 400 mm long, one multiplies by ½ to find the c^2 equivalent, 200 mm. If a g^2 string is 200 mm long, one multiplies by 3/2, i.e., 1.5 (or, if one takes temperament into consideration, some other nearby factor, such as 1.4953… for the fifth in quarter-comma meantone) to find the c^2 equivalent of 300 mm (or 299 with the meantone factor).

4. Edward L. Kottick, *A History of the Harpsichord* (Bloomington: Indiana University Press, 2003), 18, cites my draft of an unpublished study in which, seen from the perspective of the lower notes, this type of scaling is called "rising." Fortunately, this term has not come into common use.

5. William R. Thomas and J. J. K. Rhodes, "The String Scaling of Italian Keyboard Instruments," *Galpin Society Journal* 20 (1967): 48–62; Frank Hubbard, *Three Centuries of Harpsichord Making* (Cambridge, Mass.: Harvard University Press, 1965), 350.

6. Grant O'Brien, "The String-Scaling Design of Some Modern Pianos: An Introduction to the Catalogue of the Post-1850 Pianos in the Giulini Collection" and "Schedules of the Modern Pianos," in John Henry van der Meer et al., *Alla ricerca dei suoni perduti: Arte e musica negli strumenti della collezione di Fernanda Giulini* (Briosco: Villa Medici Giulini, 2006), 282–367.

"foreshortened"/"foreshortening" for Hubbard's sense. Since "non-pythagorean" could conceivably refer to other types of scaling, for example, the falling-off in the top octave occasionally found in harpsichords (sometimes explicable as resulting from the difficulty of bending the bridge to a tight curve), this term would best be used generically for any type of scaling in which the octaves are wider or narrower than those with the 2:1 ratio. It thus would best not be used for that specific type which we are now calling tapered scaling. Another terminological refinement is that one might occasionally use the term "stretched" with reference to scalings which are mainly pythagorean but have a few relatively longer notes at the top of the compass. This can happen, for example, in late French harpsichords with four registers in which, to span the gap, the 4´ bridge must be placed a little farther back than it normally would be.

In organs, in which the various speaking lengths within a rank of pipes are necessarily pythagorean (or nearly so), scaling concerns the diameters of the pipes.[7] The two extreme cases are, on the one hand, scales in which all the pipes are the same diameter and, on the other, ones in which the pipe diameters halve at the octave, that is, are determined according to the pythagorean octave ratio 2:1. The first, constant-width scaling, was generally the case until the late Middle Ages,[8] but as compasses became larger than one or two octaves, it was found that the low-pitch pipes, relatively narrow in the sense of their width-to-length ratio, sounded too soft and "stringy" if they could be made to speak at all, while the relatively wide upper pipes were comparatively too loud and "fluty." The second extreme case, pythagorean width scaling, is also impracticable, as the low-pitch pipes would sound relatively too loud and "tubby," and the high-pitch pipes would sound too weak and thin, if not being too narrow to be made at all. Thus, systems were developed in which the width-to-length ratio varied more moderately than in constant-width scaling, and in which pipes were relatively narrower in the bass and wider in the treble than with pythagorean width scaling.

7. A useful survey of historical organ-pipe scaling is Christard Mahrenholz, *Die Berechnung der Orgelpfeifen-Mensuren vom Mittelalter bis zur Mitte des 19. Jahrhunderts* (Kassel: Bärenreiter, 1938). See also Poul-Gerhard Andersen, *Organ Building and Design*, translated by Joanne Curnutt (New York: Oxford University Press, 1969), 39–51.

8. For example, a set of nearly 220 organ pipes, all 28 to 29 mm in diameter, from the eleventh or early twelfth century, excavated near the Church of the Nativity in Bethlehem (now in the Studium Biblicum Franciscanum Museum in Jerusalem), is described in Jeremy Montagu, "Bethlehem Organ of Latin Kingdom Date," in Linda Bunneghem, ed., *Liber Amicorum for Jeannine Lambrechts-Douillez on the Occasion of her 80th Birthday* ('sGravenwezel, Belgium: Valentin Lambrechts, 2008), 60–65.

Analogies between scaling organ-pipe widths and scaling string lengths are obvious. The constant lengths of lyre strings or the open strings of violins and guitars are analogous to constant-width pipe scales. The treble strings of most harpsichords, or even far into the bass in Italian instruments, are, like the lengths of a rank of organ pipes, pythagorean, as are the lengths obtained by stopping strings on fingerboards. Many pianos, including the MHS piano, and some harpsichords have tapered scales, analogous to the width scaling of virtually all ranks of organ pipes since the Middle Ages.

Key Notes and Methods of Scaling Organ-Pipe Widths

The organological literature over the past several decades has established that early stringed-keyboard instrument makers, in scaling an instrument and marking the position of its bridge, established the length of one note in each octave.[9] (In the following discussion, note names of a certain pitch class, that is, not in any particular octave, are written as italic upper-case letters.) This "key note," as I have called it,[10] was usually measured in integral or simple fractional units (½, occasionally perhaps ¼) of the maker's particular local unit of measure—inch, *duim, Zoll, pulgada, dedo, once*, etc., that is, standard divisions of the local foot, *voet*, *Fuß*, *pied*,

9. Historical sources include the plan showing the *C*-strings and their lengths in Fabio Colonna, *La Sambuca Lincea overo dell'istromento musico perfetto libri iii* (Naples, 1618), 77 (reproduced in Lynn Wood Martin, "The Colonna-Stella Sambuca Lincea, an Enharmonic Keyboard Instrument," this JOURNAL 10 (1984): 5–21, at 9), and the lists of *C*-string lengths of various models of Netherlandish harpsichords in Claas Douwes, *Grondig Ondersoek van de Toonen der Musik* (Franeker, 1699), 106. Modern studies discussing this custom include Herbert Heyde, *Musikinstrumentenbau 15.–19. Jahrhundert: Kunst, Handwerk, Entwurf* (Leipzig: Deutscher Verlag für Musik, 1986), 161–62 and 164; Grant O'Brien, *Ruckers: a Harpsichord and Virginal Building Tradition* (Cambridge: Cambridge University Press, 1990), 106; Wilson Barry, "The Scaling of Flemish Virginals and Harpsichords," this Journal 17 (1991):115–135 (albeit with the questionable proposal that the F strings were measured in addition to the C strings); Denzil Wraight, "The Stringing of Italian Keyboard Instruments c.1500–c.1650" (Ph.D. thesis, The Queen's University of Belfast, 1996; revised 1997), 121–22 and 187–88; John Koster, "Three Early Transposing Two-Manual Harpsichords of the Antwerp School," *Galpin Society Journal* 57 (2004): 81–116 and "The Early Neapolitan School of Harpsichord Making," Domenico Scarlatti en España / Domenico Scarlatti in Spain, Luisa Morales, ed. (Garrucha, Almería, Spain: Asociación Cultural LEAL, 2009), 47–80.

10. See John Koster, "Traditional Iberian Harpsichord Making in Its European Context," *Galpin Society Journal* 61 (2008): 3–78.

palmo, *braccio*, etc. Sometimes, the key-note positions were supplemented by marks for an additional note in the middle of each octave, but, as in the practice of the Ruckers family, discussed below, these lengths were typically not measured in simple units. In instruments with *F*-oriented compasses, such as sixteenth-century Venetian harpsichords and virginals and five-octave French and English harpsichords, makers evidently used *F* as the key note.[11] The MHS piano, however, with its compass beginning and ending on *C*, is clearly from the tradition of *C*-oriented compasses, for which the *C* in each octave was the key note. *C*-oriented compasses, already common in sixteenth-century Flemish instruments, became almost universally prevalent in the seventeenth century and coexisted alongside *F*-oriented compasses during the eighteenth. In this context, a particularly significant example of a *C*-oriented scaling is in the plan of a clavichord (fig. 1), compass C to c^3, reliably attributed to David Tannenberg (1728–1804),[12] who had learned the craft of building organs, presumably also stringed keyboards, from John Clemm. Only the five C strings are shown (plus B-natural, with which begins a wider spacing for the thicker strings of the lowest octave), and the lengths of the highest three are explicitly labeled *20*, *10*, and *5 Zoll*.

Builders of organs with *C*-oriented compasses similarly used scalings determined by the dimensions of *C* pipes, specifically their circumferences, that is, the widths of the flat metal plates from which the cylindrical pipes were formed. The usual starting point of organ-pipe scaling was the lowest note of the compass. (Before the prevalence of *C*-oriented compasses, this was often B-natural in the fifteenth century and F in the sixteenth and early seventeenth.) In some scaling systems, such as that used for most ranks of flue pipes in Dom François Bedos de Celles, *L'Art du facteur d'orgues* (Paris, 1766–1778), no specific attention was given to the higher *C* pipes: the plate widths of all the pipes of the rank were generated by a chart in which the widths were related to low C by Pythagorean ratios to which a constant was added. In fig. 2, adapted from Bedos's chart for 8´ and

11. Regarding Venetian instruments, see Wraight, loc. cit.

12. See Thomas McGeary, "David Tannenberg and the Clavichord in Eighteenth-Century America," *Organ Yearbook* 13 (1982): 94–106. Laurence Libin, "New Insights into Tannenberg's Clavichords," in B. Brauchli et al., eds., *De Clavicordio VII: Proceedings of the VII International Clavichord Symposium, Magnano, 7–10 September 2005* (Magnano: Musica Antiqua a Magnano, 2006),129–55. Libin, "The Memoirs of David Tannenberg," *Journal of Moravian History* 2 (Spring 2007): 118–34, noting that this is likely the drawing in a 1766 inventory.

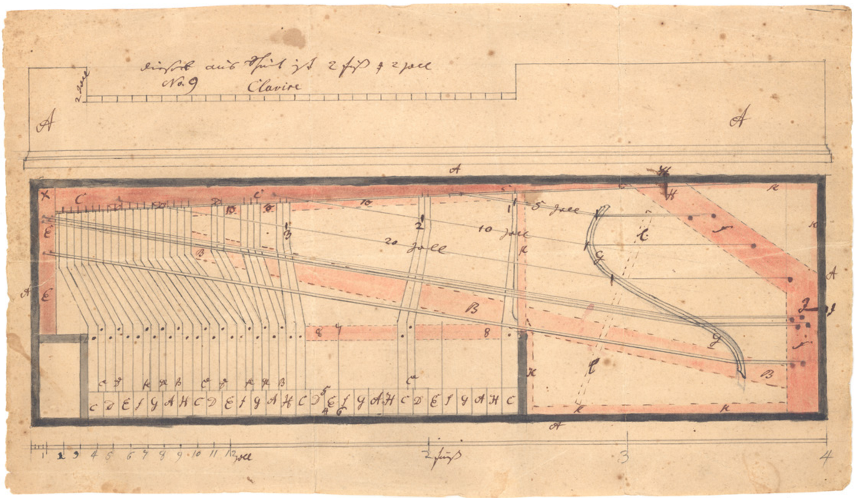

FIGURE 1. David Tannenberg's plan drawing of a clavichord, about 1780. Courtesy of the Moravian Music Foundation, Winston-Salem, North Carolina.

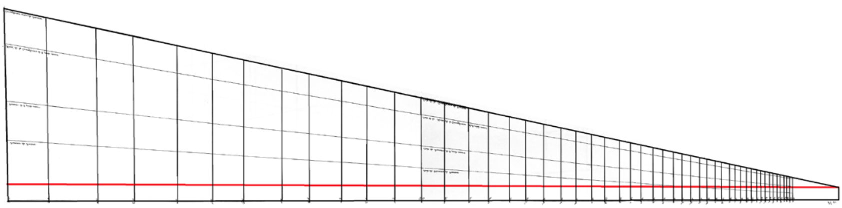

FIGURE 2. Scaling chart for 8´ and 4´ Principal ranks, adapted from Dom François Bedos de Celles, *L'Art du facteur d'orgues* (Paris, 1766–1778), plate 21, with added red line. The portion of the pipe plate widths above the line is Pythagorean (octave ratio 2:1), to which is added the the constant width below the line.

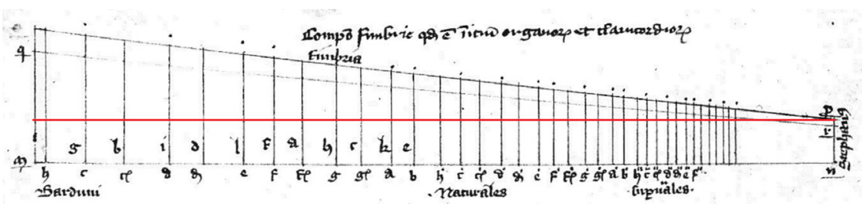

FIGURE 3. Scaling chart (*fimbria*) by Henry Arnault de Zwolle, Dijon, about 1440 (Paris, Bibliothèque nationale de France, ms. lat. 7295, fol. 129), with red line added to indicate the division between the Pythagorean portion of the pipe plate widths (above) and the addition constant (below).

4´ Principal ranks, the plate widths for each pipe are represented by the vertical lines from the baseline to the slanted line at the top. The portions above the red line follow Pythagorean scaling; the portion below the line is the addition constant, which presumably was determined empirically. The positions for the notes on the baseline, from which the vertical lines rise, were determined by dividing the baseline as if it were a monochord: c, an octave above the low C, falls at half the distance from the right end of the baseline, c^1 at a quarter the distance, c^2 at an eighth, and so on. This and others of Bedos's scaling charts are remarkably similar to that drawn about 325 years earlier by Henry Arnault de Zwolle (fig. 3). Even if here the addition constant, the portion below the red line, is much larger than Bedos's, the underlying principles are the same, including the nearly identical division of the notes by unequal semitones generated by a series of Pythagorean ratios. The scaling chart, titled *Pfeifflin zur Chormaß*, for a set of pitch pipes in Michael Praetorius's *De Organographia (Syntagma musicum* 2; Wolfenbüttel, 1619), 232, although covering only one octave, is also of the same type. Traditions of scaling and design could persist for centuries, from the time of Dufay to the time of Mozart.

In a parallel tradition, the widths of all the *C*-pipes were specifically determined as geometric progressions with octave ratios other than the Pythagorean 1:2. The *C*-widths were used to construct a chart from which the widths of the notes between the *C*-pipes could be measured. Specific historical examples include the system specified in Claas Douwes's *Grondig Ondersoek van de Toonen der Musik* (Franeker, 1699) in which, beginning with low C, the width of each successive *C* is three-fifths the width of the previous. The same system, illustrated with a chart (fig. 4), is also shown in Jan van Heurn, *De Orgelmaaker* (Dordrecht, 1804–1805). Along with the 3:5 octave ratio, Georg Andreas Sorge's *Der in der Rechen- und Meßkunst wohlerfahrne Orgelbaumeister* (Lobenstein, 1773), p. 9, mentions 4:7, 5:9, and 5:8. Already in 1434, a method resulting in an octave ratio of 3:4 was described in a treatise by Georgius Anselmi.[13] The factor of 1 to the square root of 2 is found in several seventeenth-century sources, such as Athanasius Kircher's *Musurgia universalis* (Rome, 1650), where it is presented as a set of circles inscribed inside and outside nested squares alternately placed at 45 degrees to each other (Figure 5). This method

13. See Klaus-Jürgen Sachs, *Mensura fistularum: Die Mensurierung der Orgelpfeifen im Mittelalter*, 2 vols. (Stuttgart [vol. 1] and Murrhardt [vol. 2]: Musikwissenschaftliche Verlags-Gesellschaft, 1970 and 1980), vol. 1, 142–44 and 220–23; and vol. 2, 83–86.

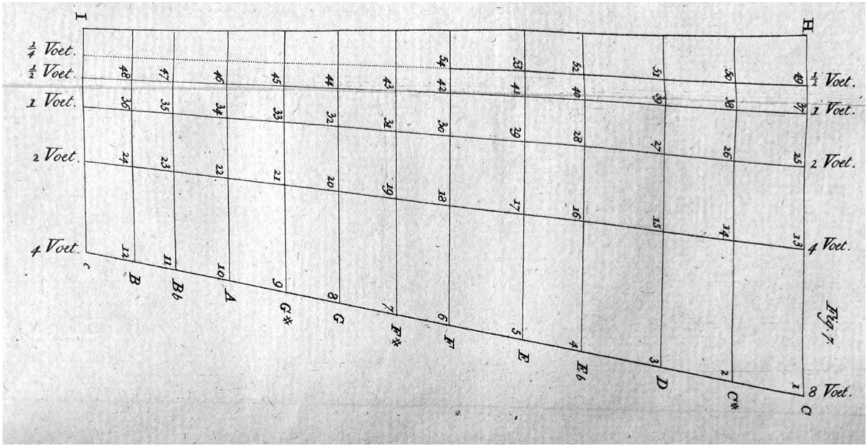

FIGURE 4. Scaling chart from Jan van Heurn, *De Orgelmaaker* (Dordrecht, 1804–05), plate 22. The distances between the baseline, here at the top, and each slanted line below represent the widths of an octave of pipes. The widths of the *C* pipes, along the outer lines, are in the octave ratio 5:3.

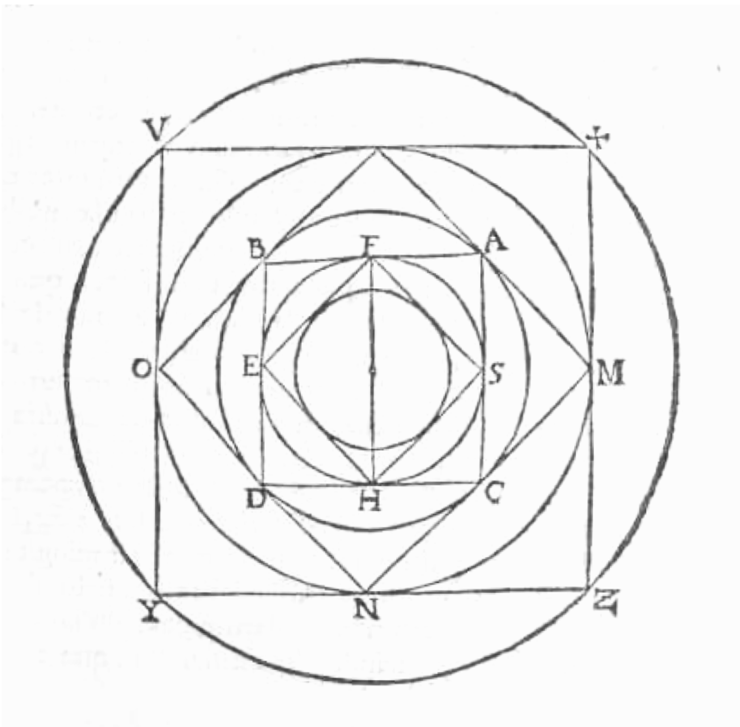

FIGURE 5. A method of scaling pipes with the octave factor of 1 to the square root of 2: a set of circles inscribed inside and outside nested squares alternately placed at forty-five degrees to each other, from Athanasius Kircher, *Musurgia universalis* (Rome, 1650), vol. 1, 511.

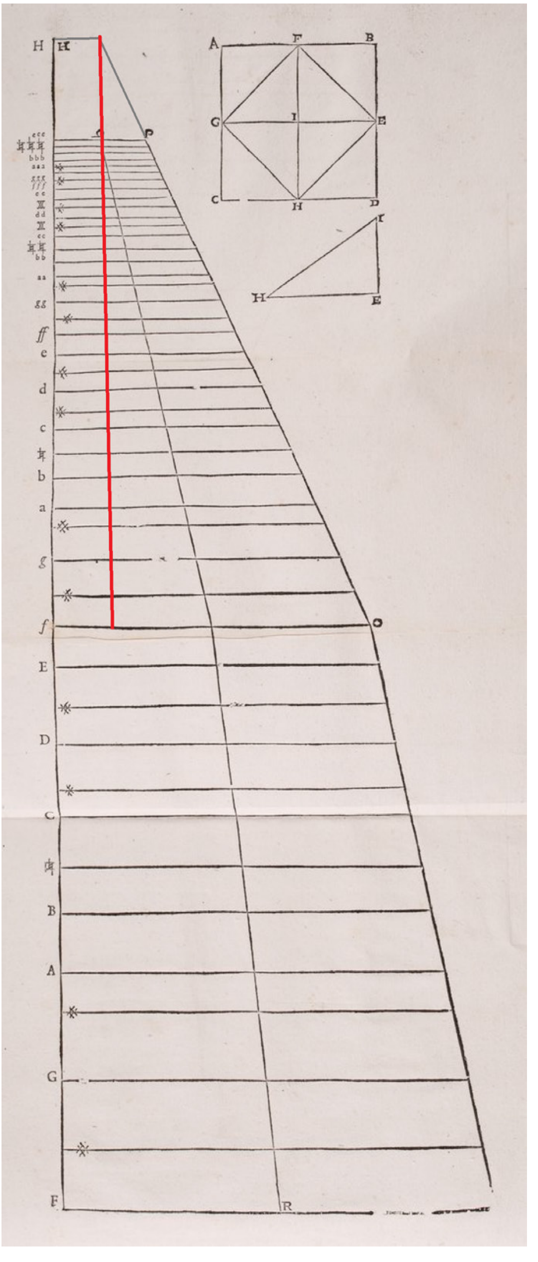

FIGURE 6. Scaling chart from Salomon De Caus, *Les Raisons des forces mouvantes* (Frankfurt am Main, 1615), with its baseline at the left. From f to c^3, the portions of the plate widths to the right of the added red line are pythagorean, with the addition constant to the left. In the lowest octave, the lowest note, F, is the full width of f, times the square root of two. The nested squares at the upper right are a version of the method shown in fig. 5 for calculating this factor with a geometrical construction.

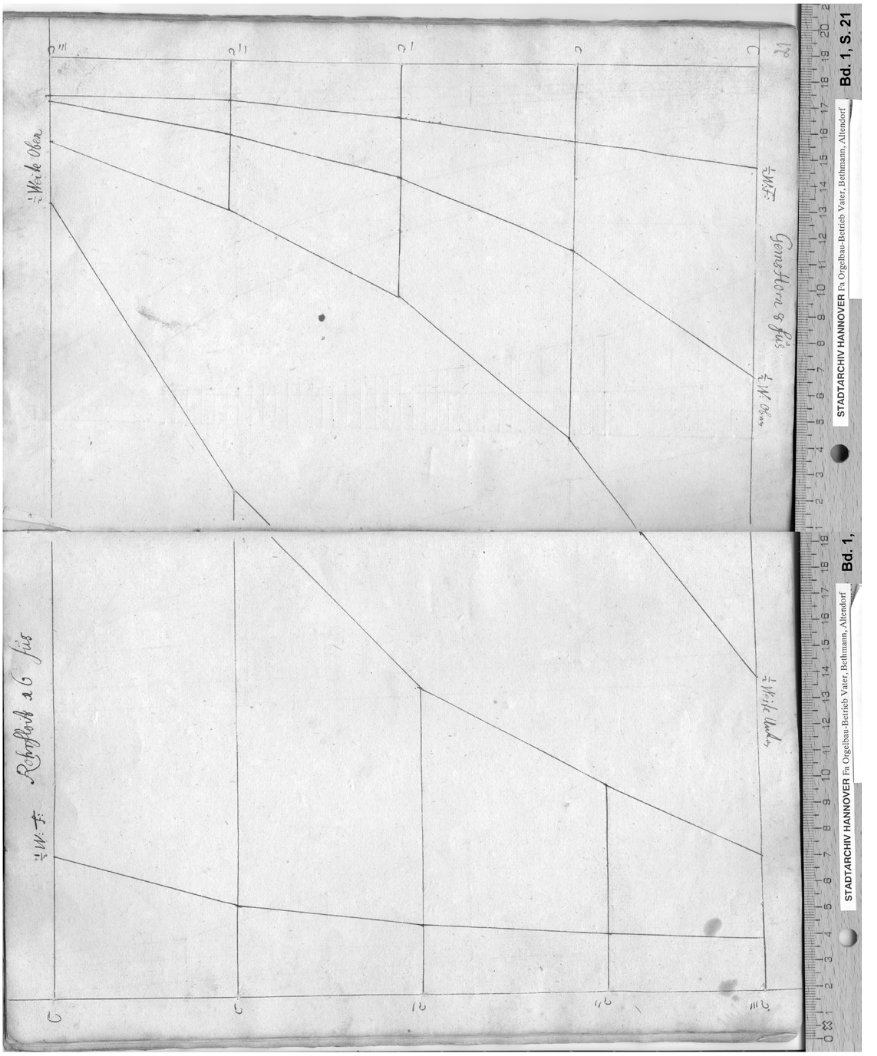

FIGURE 7. Christian Vater's scaling chart for a *Rohrfloit* 16´ and, above to the right, a *GemsHorn* 8´ (*Werkstattbuch*, 20–21). The upper slanted lines of the *Rohrfloit* chart are for half the plate widths (circumferences) of the pipes. The lines just above the baseline indicate half the plate widths of the pipe toes. In the chart for the *GemsHorn*, a rank of conical pipes, the three sets of slanted lines indicate half the plates widths at the tops and bottoms of the speaking portion, and half the widths of the toes. The vertical lines in both charts indicate the several *C* pipes. Photo courtesy of Stadtarchiv Hannover.

might have been used for organ pipe scaling much earlier, as squares so nested were commonly used to determine the sizes of elements in medieval architecture.[14]

Nineteenth- and early-twentieth-century treatises on piano design advocate tapered scalings with such octave ratios as 8:15, 8:15½, and 9:17.[15] Even before this practice was codified in theory, however, pianos were being made with tapered scalings based on a constant octave ratio. Michael Latcham, analyzing various late-eighteenth- and early-nineteenth-century pianos, has determined, for example, that several makers used tapered scalings with an octave ratio of 1:1.95.[16] The earliest of these was Johann Andreas Stein, who, trained as an organ builder, might have taken the idea from that craft.

In organ building, there were also bent scales, i.e., ones in which the octave ratio, or other basis for scaling, changes in one or more places. An example from Salomon De Caus, *Les Raisons des forces mouvantes* (Frankfurt am Main, 1615) is shown in fig. 6. Here the pipe widths upwards from f were constructed in a manner similar to that of Henry Arnault and Dom Bedos, pythagorean with the addition constant here to the left of the added red line. In the lowest octave, however, the width of the lowest note, F, is the full width of f times the square root of two. The nested squares at the upper right show the method (similar to Kircher's in fig. 5) for finding this length geometrically.

An important document regarding scaling in the period of the MHS piano is the workshop book of the north-German builder Christian Vater.[17] Vater (1659–1756) began this book, consisting mainly of scaling

14. See Lon R. Shelby, *Gothic Design Techniques: The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns Schmuttermayer* [edition of the original texts, translation, and commentary] (Carbondale and Edwardsville: Southern Illinois University Press, 1977). An earlier example is in the thirteenth-century sketchbook of Villard de Honnecourt (Paris, Bibliothèque Nationale, ms. fr. 19093, fol. 20). One should note, however, that Mahrenholz, *Die Berechnung der Orgelpfeifen-Mensuren*, 40, evidently unaware of Henry Arnault's scaling chart but relying on measurements of the pipe diameters in his drawing of an organ façade as reproduced in a secondary source, erroneously regarded these as generated by the octave factor of 1 to the square root of 2.

15. These ratios are mentioned, for example, in S. Wolfenden, in *A Treatise on the Art of Pianoforte Construction* (London, 1916), 166–67.

16. Michael Latcham, *The Stringing, Scaling and Pitch of Hammerflügel Built in the Southern German and Viennese Traditions*, 2 vols. (Munich and Salzburg: Musikverlag Katzbichler, 2000).

17. Facsimile edited by Uwe Pape as *Das Werkstattbuch des Orgelbauers Christian Vaters* (Berlin: Pape Verlag, 2001). The original is in the Stadtarchiv, Hannover, in the collection Fa Orgelbau-Betrieb Vater, Bethmann, Alterndorf.

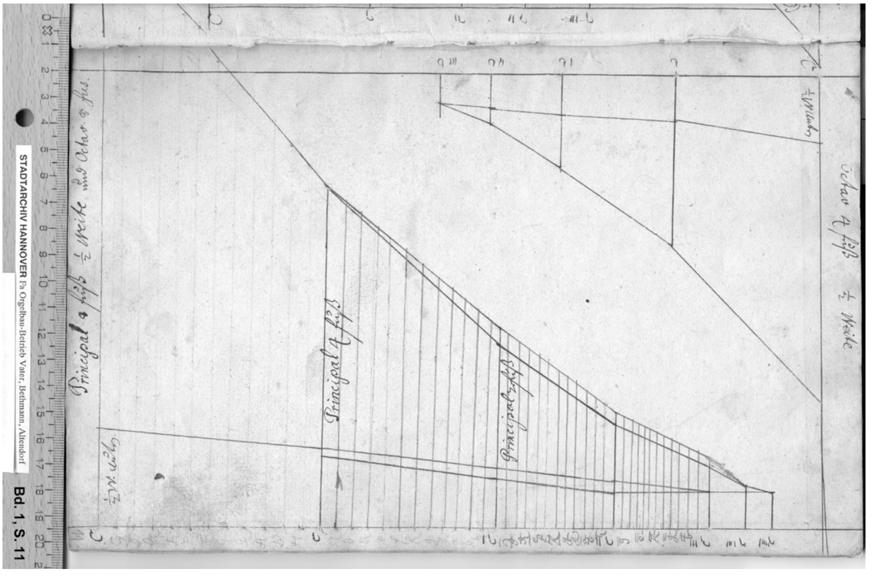

FIGURE 8. Christian Vater's scaling chart for Principals 8´, 4´, and 2´ (*Werkstattbuch*, 11). The upper slanted lines are for half the plate widths of the 8´ rank, while the lines just below this are for the somewhat narrower pipes of the 4´ and 2´ ranks. The vertical lines marking the notes between the C pipes should, in theory, be spaced somewhat closer together for each higher note, but Vater has spaced them evenly in the faint marks for the lowest octave and haphazardly in the upper octaves. Photo courtesy of Stadtarchiv Hannover.

diagrams for various stops and tables of mixture compositions, in 1697, around the beginning of his five years of working in Hamburg for the leading north-German builder, Arp Schnitger. His charts (as for the *Rohrfloit* 16´ and *GemsHorn* 8´ shown in fig. 7) typically consist of lines, perpendicular to the base line, with the plate widths (usually, because of limited space on the page, a half of this measurement) of all the *C* pipes, the upper ends connected by slanted straight lines. Rather than containing all the octaves in a compact chart like van Heurn's, with the several *C* to *C* octaves stacked above each other, Vater placed the octaves side by side. In Vater's charts, the ratio from octave to octave varies considerably. The pipe widths were presumably determined empirically by experience or by variable additions to widths determined by the Pythagorean 2:1 octave ratio, or perhaps other ratios or irrational factors.

Two of Vater's diagrams include lines for the plate widths of the eleven notes between the *C* pipes, but the lateral spacing, unlike the carefully graded divisions in van Heurn's diagram (also in the diagrams of Henry Arnault, Dom Bedos, and others), is haphazard, as, for example, in his diagram for Principals 8´, 4´, and 2´ (fig. 8). Claas Douwes was equally casual with the notes between the *C* pipes. He wrote, "When the first, second, third, fourth, and fifth C are divided in this way [i.e., with his 3:5 octave ratio], the widths of the other pipes from one C to the other are easily divided, as the distance is not large and a little bit does not matter."[18] If, however, as discussed below and shown in fig. 13, the maker of a stringed-keyboard instrument were to draw the curve of a bridge with string lengths determined so haphazardly, the line would be noticeably bumpy.

As mentioned above, organ-pipe width scales are of the same ilk as tapered string scales, but from the traditional organ builder's perspective of beginning from the bottom of the compass they might better be regarded as widening than as tapering. For both stringed keyboards and organs, however, a mixed perspective is conceivable. In the former, the alternative c^2 lengths of 15 and 14 *duimen* found in sixteenth-century Flemish virginals presumably tuned to pitches a quarter-comma-meantone diatonic semitone apart, an interval with the (closely approximate) ratio 15:14, would seem to suggest that c^2 was of particular significance in the makers' procedure.[19] The importance of this note in the design process is also suggested by O'Brien's "49 cm rule" according to which the 8´ bridge pin for the string sounding c^2 in Ruckers harpsichords is 49 cm or 19 *duimen* from the nameboard.[20] The use of c^2 as the principal note for scaling would also help to bypass difficulties in calculating with c^3 lengths affected by the falling-off in the top octave mentioned above.

In organ building, De Caus's bent scale chart was clearly generated from the pipe for f, up to c^3 in one direction and down to F in the other. This mixed perspective seems to have become pervasive in Germany during the seventeenth century. Andreas Werckmeister noted in his *Orgel-Probe* (Quedlinburg, 1698) that organ builders "care to take something from the width of the low or large notes and, in return, give something

18. Douwes, *Grondig Ondersoek*: "Wanneer de 1 2. 3. 4. en 5, de C aldus afgedeelt sijn / so is de wijdte van de andere pijpen / van de eene C tot de andere / licht af te deelen / alsoo de spatie niet groot is / ende het op een weinigje niet aan komt."

19. The alternative 14- and 15-*duim* scalings are discussed in John Koster, "The Virginal by Hans Bos, Antwerp, 1578, at the Royal Monastery of Santa Clara, Tordesillas," in *Música de tecla en los monaterios femeninos de España, Portugal y las Américas*, ed. Luisa Morales, (Garrucha, Almería, Spain: Asociación Cultural LEAL, 2011), 67–89.

20. O'Brien, *Ruckers*, 106.

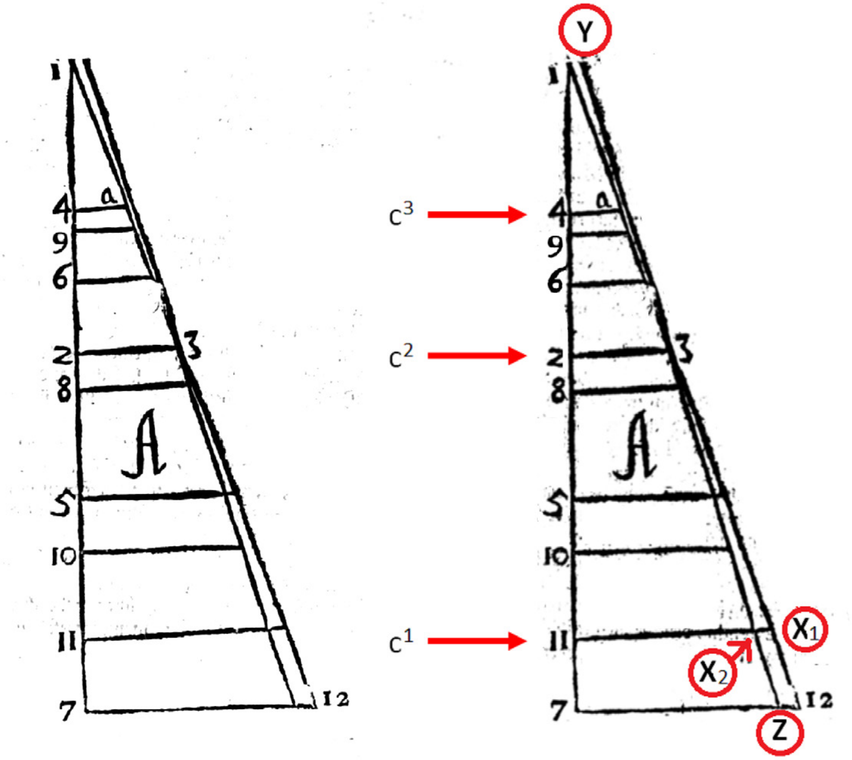

FIGURE 9. Left: scaling chart A from Johann Philipp Bendeler, *Organopoeia* (Frankfurt and Leipzig, 1690), 9. Right: the same with additional labels for clarification.

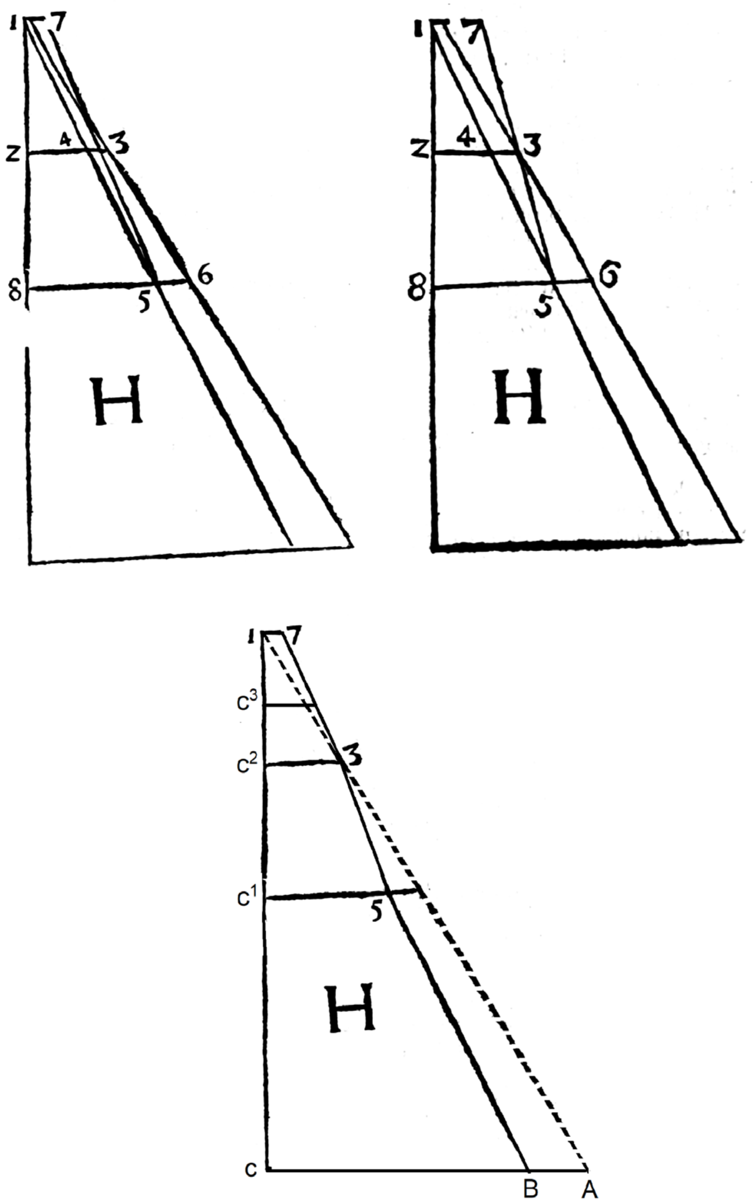

FIGURE 10. Above left: scaling chart H from Johann Philipp Bendeler, *Organopoeia* (Frankfurt and Leipzig, 1690), 16. Above right: the unsatisfactorily corrected version in the second edition, retitled *Orgel-Bau-Kunst* (Frankfurt, 1739), 20. Below: the present author's corrected adaptation, simplified for clarity.

to the smaller pipes."[21] Although the result is similar to those of the Pythagorean-plus-constant system of Henry Arnault, Dom Bedos, et al., or of the octave-ratio system of Douwes, van Heurn, et. al., both of which begin with the largest pipe, the perspective of Werckmeister's statement is from the middle, looking down to the lower pitches and up to the higher.

The mixed up-and-down perspective was made explicit in Johann Philipp Bendeler's *Organopoeia* (Frankfurt and Leipzig, 1690), which describes the scaling of a rank of pipes beginning with a "known pipe" (p. 9: *bekandten Pfeiffe*) of the desired pitch and tone quality. Bendeler's scaling chart *A* (fig. 9) and the instructions associated with it were evidently intended more to explain his method than to specify the construction of any particular rank of pipes. The compass, represented by the pythagorean division of pipe lengths in its baseline, which is placed vertically, is not fully worked out but extends only one octave above the known pipe while extending into the second octave below. Bendeler mentions that his known pipe is *C* without clearly indicating which one, but he gives its length as 206 *Scrupel* and its (plate) width, i.e., circumference, as 70 *Scrupel*. We do not know the value of Bendeler's *Scrupel*, a unit of measurement which, as a fraction of a fraction of an inch, had widely different values at different places and times.[22] Nor do we know which regional foot standard he used. The quotient of the length divided by the width of Bendeler's known pipe, however, 2.94, is consistent with a pitch of c^2.[23] Thus, the top note shown in Bendeler's chart would be c^3, then typically the top note of German organ compasses. Towards the end of his discussion, Bendeler mentions that one might choose as the known pipe one that is "large and

21. Andreas Werckmeister, *Orgel-Probe* (Quedlinburg, 1698), 20. "in der Tieffe oder grossen Stimmen der Weite etwas benehmen, und hergegen in der kleinern Pfeiffen etwas zu zugeben pflegen." According to Mahrenholz, *Die Berechnung der Orgelpfeifen-Mensuren*, 51, this was already in the first edition of *Orgel-Probe*, published in 1681.

22. According to Horace Doursther, *Dictionnaire universel des poids et mesures anciens et modernes* (Brussels, 1840), 482, in Prussia the *Scrupel* was one 1728th of a foot (one 144th of an inch), while in some other German regions it was one 144th of a foot (one twelfth of an inch). The article on *Scrupel* in Johann Heinrich Zedler's *Universal Lexicon* 36 (Leipzig, 1743) mentions a "geometric" system of length measurement in which the *Fuß* was divided into ten *Zoll*, this into ten *Grad*, and this into ten *Scrupel*, each of which, therefore, was one-thousandth of an inch. Sorge, in *Der in der Rechen- und Meßkunst wohlerfahrne Orgelbaumeister*, p. 11, likewise divided the length of his c^2 pipe, nominally one foot long, from which all others were calculated, into 1000 *Scrupel*. For a reasonable pipe length and diameter, Bendeler's *Scrupel* would not have been much smaller than one 225th of a foot.

23. Andersen, *Organ Building*, p. 45, regards Bendeler's known pipe as c. The length/width quotient for Principal pipes sounding c in historical organs, however, is generally in the range of 4 to 4.5.

nearing the deep pitches."[24] All in all, however, we might well gather that he regarded c^2 as the usual pitch of the known pipe. This certainly was the case for G. A. Sorge, who recommended that one begin with the c^2 (one foot) pipe of the 8' Principal, such that (in almost a paraphrase of what Werckmeister had written) "the pipes upwards from c^2 gain something in width, and downwards from c^2 they lose."[25] Accordingly, I have marked c^2 and other notes on Bendeler's chart. One should mention that the numbers in the chart do not designate measurements or ratios, but only points mentioned in his exposition, to which I have added letters to indicate other points of significance.

Starting with the known pipe (c^2), its length is that between points 1 and 2 on the vertical baseline and its plate width between points 2 and 3. A line is drawn from point 1 through point 3 and extended to point 12. The length 1 to 4 is for the higher octave (c^3) and 1 to 11 for the lower octave (c^1). The lengths of other notes, found by a succession of ratios, are marked at other points (5, 6, 7, 8, 9, and 10) along the vertical baseline. (Bendeler did not mark all the notes in his chart but went through all the calculations in his text.) The horizontal lines from the vertical line to the slanted line, drawn from point 1 through point 3 to point 12, represent the plate widths of the pipes according to pythagorean scaling. This is, for example, the width of c^1 (from point 11 to point X_1) and that of c^3 (from point 4 to point a) which are, respectively, twice and half the width of c^2 (from point 2 to point 3). The actual pipes, however, were made narrower than their pythagorean widths below the known pipe and wider above. For this, a certain amount, from point X_1 to X_2, was subtracted from the width of c^1. A line was then drawn through points X_2 and 3, extending beyond them to points Y and Z. The width added to c^3, from point a to line YZ, is half the width from X_1 to X_2. The adjusted widths of all the pipes are the widths from their points on line 1–7 to line YZ.

In effect, Bendeler's method results in a chart like that of Dom Bedos, if the bass end of the horizontal baseline were moved up above the red line. Some latitude was allowed in applying the principle by which, in Werckmeister's words, one should "take something from the width of the low or large notes and, in return, give something to the smaller pipes."

24. Bendeler, *Organopoeia*, p. 14: "groß und denen tieffen Sonis nahe."

25. Sorge, *Der in der Rechen- und Meßkunst wohlerfahrne Orgelbaumeister*, 11–12: "Von c″an aufwerts gewinnen also die Pfeifen etwas an der Weite, und von c″ an abwerts verlieren sie."

The first "something" need not be equivalent to the second "something," which in Bendeler's chart was one-half the first.

Bendeler provided a second chart, labeled H (*Organopoeia*, p. 16).[26] There was, however, an error in the execution of this chart, acknowledged in a page of errata but not satisfactorily corrected in the second edition.[27] Fig. 10 shows the faulty originals together with a corrected adaptation, simplified for clarity.[28] As in chart A, the pythagorean divisions in the vertical baseline at the left of chart H represent the pipe lengths, and the known pipe is again presumably c^2. The broken slanted line from point 1 to point A represents pythagorean pipe widths, from which a certain amount is subtracted from the width of c^1, to obtain its narrowed width from the baseline to point 5. The pipe widths below c^1 are indicated by the line from point 5 to point B, which is in a straight line with point 1. The widths from c^1 to c^2 are indicated by the line from point 5 to point 3. The distance from point 1 to point 7 is one-half the width subtracted from c^1. The widths of the pipes above c^2 are indicated by the line drawn from point 3 to point 7. Thus, in the line 7-3-5-B indicating the pipe widths, there are bends at c^1 (point 5) and c^2 (point 3). The width added to the pythagorean width of the c^3 pipe is one-quarter the amount subtracted from the width of c^1.

If, then, for Bendeler the addition to the c^3 pipe width could vary from one-half to one-quarter the amount subtracted from c^1, we might gather that other fractional amounts could be admitted in practice, as also could various bends. Indeed, practice must have preceded Bendeler's theoretical systematization, for which merely two examples served his purpose. As he wrote: "one must nevertheless use his judgement, which is easy to do when one knows the proper basis of scaling."[29] Werkmeister had expressed the same sentiment more colorfully: "if, however, [scaling] be seen in the light, it is as difficult as the art of Columbus's egg."[30] That is, scaling was a

26. There are no charts B to G.

27. The page of errata is not included in the facsimile edited by Rudolf Bruhin (Amsterdam: Frits Knuf, 1972), but can be seen in the Library of Congress's copy, available online at https://www.loc.gov/item/08017339/. The second edition (Frankfurt, 1739) is also available online at https://www.loc.gov/item/09032335/.

28. In a future publication, I will discuss in detail Bendeler's chart H, which, I believe, is misinterpreted in Andersen, *Organ Building*.

29. Bendeler, *Organopoeia*, 14. "Jedoch muß man sein Judicidum gebrauchen, welches denn leicht geschehen kann, weil man das rechte Fundament der Mensuration weis."

30. Ibid., 34. "wenn es aber beym Licht besehen wird, ist es so schwer als des Columbi Eyer Kunst."

matter with practical, not necessarily elegant solutions, just as Columbus, according to legend, stood an egg on end by lightly smashing it on the table.

It cannot be without significance that both John Clemm and David Tannenberg were born and raised in Central Germany, the same cultural region in which Werckmeister, Bendeler, and Sorge wrote their treatises drawing on their experience of regional traditions of organ building.

Analyzing the MHS Upright Piano's Scaling

At first glance, the non-pythagorean scaling of the MHS piano might be taken as evidence that it was made by an inexperienced provincial artisan without professional training in instrument making. After all, the scalings of the pianos of Bartolomeo Cristofori and Gottfried Silbermann are quite accurately pythagorean in the treble, and, in the work of the former, deep into the tenor.[31] John Watson, however, concluded after his detailed examination of the MHA piano that, unlike some "homespun keyboard instruments" made in the same region in the eighteenth and early nineteenth century, it "is clearly the work of a professional instrument maker with extensive experience, skill, and judgement." The aptness of this assessment is evident if we compare the scaling of the MHS piano with those of two "homespun" Pennsylvania pianos, as graphed in fig. 11. Next to the relatively smooth curve of the MHS piano (in black), the irregularity of the others is clear. In the curve (in red) of a piano by John Huber, Northampton, Pennsylvania, about 1790 (in the collection of the Northampton County Historical and Genealogical Society, Easton, Pennsylvania), there is a dip around d^2, indicated by the red arrow.[32] The curve (in blue) of the other piano, by an unknown late-eighteenth-century maker (in the collection of The Metropolitan Museum of Art, New York, acc. no. 1987.229), is markedly erratic in the bass.[33] Indeed, for nearly an octave around c, indicated by the blue arrow, the string lengths actually

31. See the measurements in Stewart Pollens, *The Early Pianoforte* (Cambridge: Cambridge University Press, 1995), 94 and 207.

32. This piano, once owned by a certain Jacob Opp, is described in Laurence Libin, "John Huber's Pianos in Context," this JOURNAL 19 (1993): 5–37.

33. This instrument is described in Laurence Libin, "A Unique German-American Square Piano," *Early Keyboard Journal* 9 (1991): 7–20.

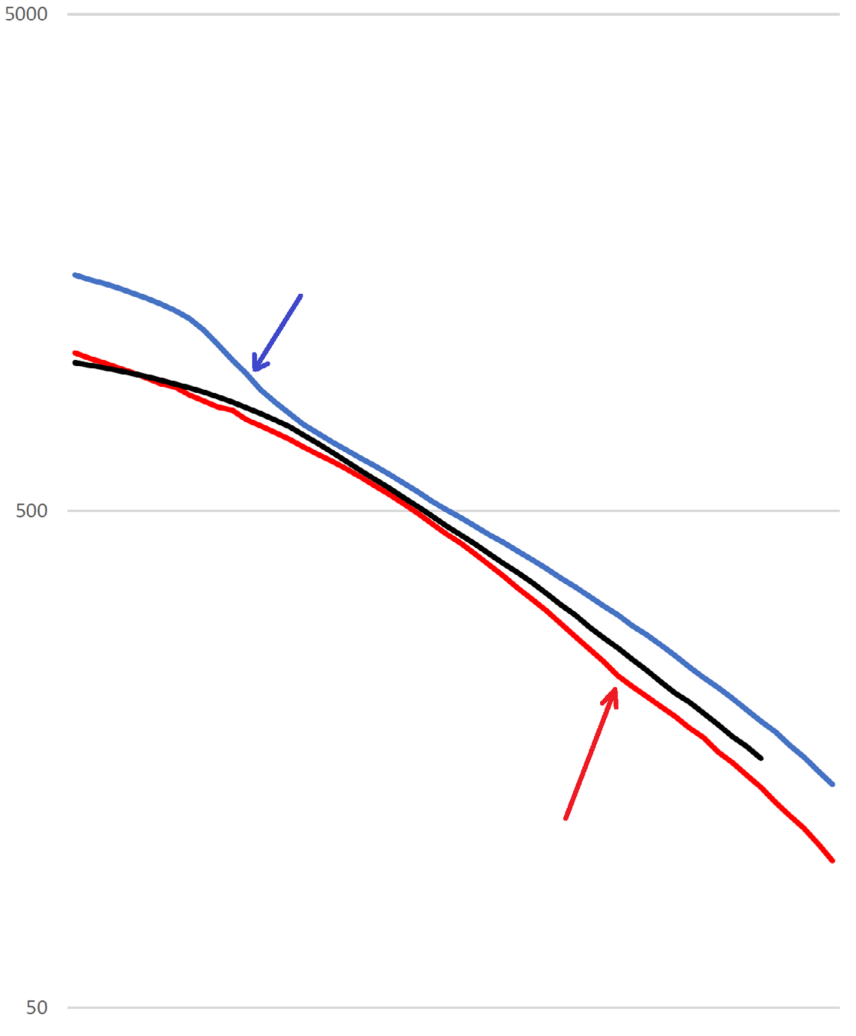

FIGURE 11. String lengths of the MHS piano and two "homespun" Pennsylvania-German square pianos, graphed with a logarithmic Y-axis. Next to the relatively smooth curve of the MHS piano (in black), the irregularity of the others is clear. In the curve (in red) of a piano by John Huber, Northampton, Pennsylvania, about 1790 (in the collection of the Northampton County Historical and Genealogical Society, Easton, Pennsylvania), there is a dip around d^2, indicated by the red arrow. The curve (in blue) of the other piano, by an unknown late-eighteenth-century maker (in the collection of The Metropolitan Museum of Art, New York, acc. no. 1987.229), is especially erratic in the bass. The slope of the curve around c (indicated by the blue arrow) shows the string lengths increasing at a rate with an octave ratio larger than 2:1. The data for the graphs are from John R. Watson's article in this issue of *JAMIS* and from Laurence Libin's articles "John Huber's Pianos in Context," *JAMIS* 19 (1993): 32–33 (column "Opp") and "A Unique German-American Square Piano," *Early Keyboard Journal* 9 (1991): 15.

increase at a rate with an octave ratio higher than 2:1. The skillful manner with which the properly tapered scaling of the MHS piano was determined deserves detailed scrutiny.

Determining the Key-Note C-String Lengths

John Watson's conclusion, that the maker of the MHS piano seems to have determined its top three *C*-string lengths as 20, 11½, and 6¼ Anglo-American inches, should be regarded as so plausible as almost certainly to represent the maker's thinking. As mentioned above, historical sources indicate that some makers, both of organs and of stringed keyboards, began with c^2. If the maker of the MHS piano did so, like his Central-German fellows Bendeler and Sorge, he would, as Watson suggests, indeed have proceeded downwards from c^2, doubling its 11½ inches, then subtracting 3; but for c^3 he would have proceeded upwards, halving the 11½ inches and adding ½. This procedure obviates the necessity of beginning at c^3 with the fractional measurement of 6¼ inches, which is rather more fussy a measurement than seems to have been typical for a key note. Starting at c^2, the procedure could be described with a slight emendation of Sorge's description of organ-pipe scaling, quoted above, substituting "strings" and "length" for his "pipes" and "width": "upwards from c^2 the strings gain something in length and downwards from c^2 they lose." Although the MHS piano was almost certainly made quite some time before Sorge's book was published in 1773, the status of c^2 as the known pipe and therefore its length as a maker's personal standard measure for that note, had likely been long-established concepts, at least in the Central German background of Moravian-American musical culture.

The length of the c^2 pipe, nominally "one-foot *C*," of an 8´ Principal rank at *Chorton*, a common pitch in German organs, approximately a semitone above a^1 = 440 hz,[34] is something less than one Anglo-American foot long. Sorge, for example, in the fifth plate of *Der in der Rechen- und Meßkunst wohlerfahrne Orgelbaumeister*, included a line the length of the standard c^2 pipe from which he calculated the rest of his scaling. In the

34. See Bruce Haynes, *A History of Performing Pitch: The Story of "A"* (Lanham, Maryland: Scarecrow Press, 2002), 461–69.

Bavarian State Library's copy of the book,[35] someone has helpfully noted in pencil that this line is 279.333 mm long, which happens to be exactly 11 Anglo-American inches. If the maker of the MHS piano were an organ builder using an 11½ inch standard for c^2 as the basis of his pipe scaling, presumably for a pitch somewhat lower than Sorge's, he might well have used this as the basis for designing stringed-keyboard instruments.

Here we might profitably turn to the earliest known existing stringed-keyboard instrument made in Dresden, a two-manual spinet (8´8´4´) in the Leipzig collection (no. 56) signed *Christophorus Heinricus Bohr Art[ifex?]: Mechanicus f[ecit] A[nno] 1713 / in Dresden*. The instrument is described in detail in Hubert Henkel's catalog of the collection, which states that Bohr is known only from this instrument.[36] Recently, however, I came across an early reference to him: Georg Menzer (1652–1711), organist of the Dom in Freiberg (Saxony) from 1694 until his death (a few months before which he was involved in commissioning the great organ by Gottfried Silbermann), "admirably learned the organ- and [stringed-keyboard] instrument-maker's art" with Christoph Heinrich Bohr in Dresden for three years after having studied for three years with the Dresden court organist, Christoph Kittel.[37] Menzer's training with Bohr must have occurred before he became organist of the Nikolaikirche in Freiberg in 1676. Since Bohr, doubtless at least a few years older than Menzer, would have been born no later than the mid-1640s, his own training as an organ and harpsichord/clavichord maker would have taken place no later than the early 1660s. Thus, he would have been steeped in the seventeenth-century organ-building practices later described and codified by such as Bendeler.

With compass GG/BB to c^3, the Bohr spinet is a *C*-oriented instrument. The lower-manual 8´ c^2 string (on the bass side of the jacks), 276 mm, or 10⅞ Anglo-American inches, is close to the length of Sorge's "known pipe" c^2 and to the c^2 string length of the MHS piano. Bohr's *C*-string lengths are shown in the Table 1. Twice Bohr's c^2 length is 552 mm, from

35. Available online at https://imslp.org/wiki/Der_wohlerfahrene_Orgelbaumeister_(Sorge%2C_Georg_Andreas).

36. Hubert Henkel, *Kielinstrumente*. Musikinstrumenten-Museum der Karl-Marx-Universität Leipzig, Katalog, Band 2 (Leipzig: Deutscher Verlag für Musik, 1979), 40–43 and 113.

37. Werner Müller, *Gottfried Silbermann: Persönlichkeit und Werk* (Frankfurt: Verlag Das Musikinstrument, 1982), 120. "die Orgel- und Instrumentenmacherkunst ehrlich gelernet."

TABLE 1. Two-manual spinet by Christoph Heinrich Bohr, Dresden, 1713 (Museum für Musikinstrumente der Universität Leipzig, Nr. 56): C-string lengths in mm.

	< 8´	4´
c^3	155	93
c^2	276	165
c^1	522	325
c	850	586
C	1098	873

Data from Hubert Henkel, *Kielinstrumente, Musikinstrumenten-Museum der Karl-Marx-Universität Leipzig, Katalog, Band 2* (Leipzig: Deutscher Verlag für Musik, 1979), 41.

which one would subtract 30 to obtain the c^1 length of 522 mm. Half the c^2 length is 138 mm, to which one would add 17 to obtain the length of c^3, 155 mm. These quantities, of subtraction from c^1 and addition to c^3 in a ratio of 1.76:1, are close to Bendeler's ratio of 2:1, especially if we consider that small discrepancies of a millimeter or so, such as would inevitably occur in positioning the bridge and nut or their pins in so complex an instrument, would have a significant effect on the precise ratio. Thus, we cannot rule out the possibility that Bohr's ideal ratio was the same as what Bendeler later described, but even if Bohr had intended some other ratio, this would have been the prerogative that Bendeler accorded a maker's judgement. In any case, one might plausibly relate Bohr's string scaling to his background as an organ builder. The practice of taking away something from the lower pipes and adding something to the small pipes would have provided the germ of an idea resulting in tapered scaling in stringed-keyboard instruments.

In the overall context of early Germanic stringed-keyboard instrument making, insofar as is known, Bohr's tapered scaling was unusual and evidently presented something new. In general, as in the harpsichords, clavicytheria, virginals, and clavichords in Table 2, German and Austrian makers aimed to make c^3 half the length of c^2, if one allows some accidental discrepancies in which the high note is a few millimeters, a quarter inch or so, longer or shorter than it should be. In most of these, the foreshortening towards the bass is already evident at c^1, as is normal in northern-European instruments, including Ruckers harpsichords. Tapered scalings are found in the earliest harpsichords for which we have detailed technical

TABLE 2. *C*-string lengths in mm of early Germanic stringed-keyboard instruments.

	C	c	c^1	c^2	c^3
Clavicytheria					
1. Anonymous, Germany, ca. 1620	1417	1065	612	340	172
2. Henning Hake, Riga, 1657	1236	1068	607	319	159
Harpsichords					
3. Johann Meyer, Stuttgart, 1619	1480	1040	555	310	160
4. Anonymous, Germany, ca. 1630	1379	1116	702	364	180
5. Valentin Zeiss, Linz, 1639	1267	1033	607	315	155
6. Valentin Zeiss, Linz, 1646	1225	951	567	293	150
7. Johann Wolffgang Schonnadt, Germany or Holland, 1643	1378	1104	652	328	170
Virginals					
8. Joos Karest, Antwerp, 1548	1164	885	497	286	139
9. V.K., Germany, 1588	1132	977	590	320	155
10. Anonymous, Germany, ca. 1600	1330	1053	640	345	153
11. Anonymous, Germany, 17th c.	1304	989	559	268	141
Clavichords					
12. Anonymous, Southern Netherlands, late-16th/early-17th-c.	871	720	459	241	125
13. Anonymous, Germany, mid-17th c.	797	635	409	225	113
14. Johann Jacob Donat, Leipzig, 1700	1000	726	445	251	114
15. Johann Christof Maywaldt, Weigandsthal, 1729	831	655	393	214	115

Locations and sources of data (if not the author's own measurements: 1. Germanisches Nationalmuseum, Nuremberg (MIR 1080). 2. Musikmuseet, Stockholm. 3. Salzburg Museum. Data from Salzburger Museum Carolino Augusteum Jahresschrift 34 (1988), 223. 4. Bayerisches Nationalmuseum, Munich (Mu 78). 5. Salzburg Museum. Data from Alfons Huber, ed., *Das Österreichische Cembalo: 600 JahreCembalobau in Österreich* (Tutzing: Hans Schneider, 2001), 502. This and the following instrument are components of claviorgana. 6. Schloss Aistersheim, Oberösterreich. Data ibid., 504. 7. Private collection, Madrid. 8. Musical Instruments Museum, Brussels. 9. Heimatmuseum, Wasserburg am Inn. Data from Fritz Thomas, "Das älteste deutsche Virginal in Wasserburg am Inn— Restaurierung und Entdeckung der Signatur," *Arbeitsblätter für Restauratoren*, 1988, Heft 2, 8–13. 10. Victoria & Albert Museum, London ("The Glass Virginal," Mus. No. 420–1872). 11. Museum für Musikinstrumente der Universität Leipzig (Nr. 49). Data from from Hubert Henkel, *Kielinstrumente*, Musikinstrumenten-Museum der Karl-Marx-Universität Leipzig, Katalog, Band 2 (Leipzig: Deutscher Verlag für Musik, 1979), 31. 12. Musical Instrument Museum, Edinburgh (no. 4486). Data from Darryl Martin, "A South Netherlandish Quint-Pitch Clavichord," *Galpin Society Journal* 69 (2016): 23–38. 13. The Metropolitan Museum of Art, New York (acc. no. 89.4.1215). 14. Museum für Musikinstrumente der Universität Leipzig. Data from Hubert Henkel, *Clavichorde*, Musikinstrumenten-Museum der Karl-Marx-Universität Leipzig, Katalog, Band 4 (Leipzig: Deutscher Verlag für Musik, 1981), 38. 15. Musical Instrument Museum of the National Museum, Poznan.

evidence: Henry Arnault of Zwolle's *clavisimbalum*, drawn as a plan about 1440, and the late-fifteenth-century upright harpsichord in the Royal College of Music, London.[38] These were apparently designed by adding a constant (in the former) or a variable (in the latter) to a Pythagorean scaling in a manner similar to that of Henry's or Bendeler's charts. The pythagorean treble scaling of the seventeenth-century Germanic instruments indicates that there was a historical gap, such that there was no connection between the fifteenth-century tapered scalings and later ones, such as Bohr's.[39]

One could argue that Bohr's augmentation of the scaling at the top of the compass was an aberrant stretching of the string lengths (reaching a c^2-equivalent of 372 mm at c^3 of the 4´), so that they could span the three sets of jacks. The spinet layout is indeed rather constricted in this area. This is no great problem in harpsichords, yet we find tapered scaling in the next-earliest known Dresden instrument, a harpsichord made in 1722 by Johann Heinrich Gräbner the Elder, organ builder to the Saxon court.[40] String lengths of this instrument (in the Villa Bertramka, Prague) are given in Table 3. In this instrument, the length of 8´ c^3 is half the 348 mm length of c^2, plus 12 mm, and c^1 is twice c^2, minus 84 mm. In Saxon measure, with one *Fuß* equivalent to 283.2 mm,[41] c^2 would be 14¾ *Zoll*, from which c^1 and c^3 would be calculated so:

for c^3: $14\frac{3}{4} \div 2 + \frac{1}{2} = 7\frac{7}{8}$

for c^1: $14\frac{3}{4} \times 2 - 3\frac{1}{2} = 26$

It is possible, however, that we should regard the compass, exceptionally,

38. I discuss Henry Arnault's scaling in "From *Fimbria to Clavisimbalum*: The Genesis of Henry Arnault's Harpsichord Plan," *Informazione Organistica e Organologica*, terza serie, n. 2; anno xxxiii, no. 48 (2021): 35–73. The instrument in London is described in Elizabeth Wells, ed., *Royal College of Music Museum of Instruments, Catalogue Part II: Keyboard Instruments* (London: Royal College of Music, 2000), 18–25. I discuss the scaling of both instruments in Koster, "Some Remarks on the Relationship Between Organ and Stringed-Keyboard Instrument Making."

39. Also presumably unrelated to German practice are the tapered scalings found in many English virginals, as described and analyzed in Darryl Martin, "The English Virginal" (Ph.D. thesis, University of Edinburgh, 2003), vol. 1, 47–54.

40. The relevance of Gräbner's tapered scaling as precedent for that of the MHS piano was first noted by Stephen Birkett to John Watson. The several existing Gräbner harpsichords are described in John Phillips, "The 1739 Johann Heinrich Gräbner Harpsichord—an Oddity or a *Bach-Flugel*?," in Christian Ahrens and Gregor Klinke, eds., *Das deutsche Cembalo: Symposium im Rahmen der 24. Tage Alter Musik in Herne 1999* (Munich and Salzburg: Musikverlag Katzbichler, 2000), 123–39, from which the measurements mentioned in the present article have been taken.

41. See Doursther, *Dictionnaire,* 407, listed under Dresden.

TABLE 3. Harpsichord by Johann Heinrich Gräbner the Elder, 1722 (Villa Bertramka, Prague): String lengths in mm.

	< 8´	4´
d^3	163	80
c^3	186	87
c^2	348	170
c^1	612	324
c	1060	603
C	1702	987
FF	1931	1208

Data from John Phillips, "The 1739 Johann Heinrich Gräbner Harpsichord—an Oddity or a *Bach-Flugel*?," in Christian Ahrens and Gregor Klinke, eds., *Das deutsche Cembalo: Symposium im Rahmen der 24. Tage Alter Musik in Herne 1999* (Munich and Salzberg: Musikverlag Katsbichler, 2000), 123–39.

as *D*-oriented. This is not implausible in that d^3 is the highest note of the compass. Moreover, a harpsichord by J. H. Gräbner the Younger, 1739, has the compass DD to d^3. Thus, the elder Gräbner might have conceived his scaling with much rounder numbers on the basis of the d^2 length of 13 *Zoll*:

for d^3: $13 \div 2 + ½ = 7$

for d^1: $13 \times 2 – 3 = 23$

In either case, whether the elder Gräbner used *C* or *D* as his key note, one cannot help but notice that his length adjustments for c^3 or d^3 (add ½ *Zoll*) and for c^1 or d^1 (subtract 3½ or 3 *Zoll*) are strikingly similar to those of the MHS piano.[42]

Below c^1, the scaling of the MHS piano exhibits the foreshortening typical in stringed-keyboard instruments. The lengths of c and C are respectively 31.79 and 39.15 inches long. One might well assume that the maker intended these to be the nice, even numbers 32 and 39. As can be seen in Table 4, the measurements of the c^1, c, and C strings

42. I might further observe that the 4´ strings are approximately a semitone's length shorter than the 8´ strings of the same sounding pitch. For example, the 4´ c^1 length of 324 mm divided by the 8´ c^2 length of 348 mm is 0.93, which is close to 0.94 (the factor for an equally tempered semitone) or to 0.9333… (the factor for a 14:15 semitone). This practice, also seen in harpsichords of other schools, might be related to Werckmeister's recommendation, in *Orgel-Probe*, 21, that higher-pitch stops be scaled somewhat narrower than the foundation stops. This can be seen in Christian Vater's scaling chart for Principal ranks (fig. 8), in which there are two sets of slanted lines at the top of the chart, the upper for the widths of the 8´ rank, the lower for those of the 4´ and 2´.

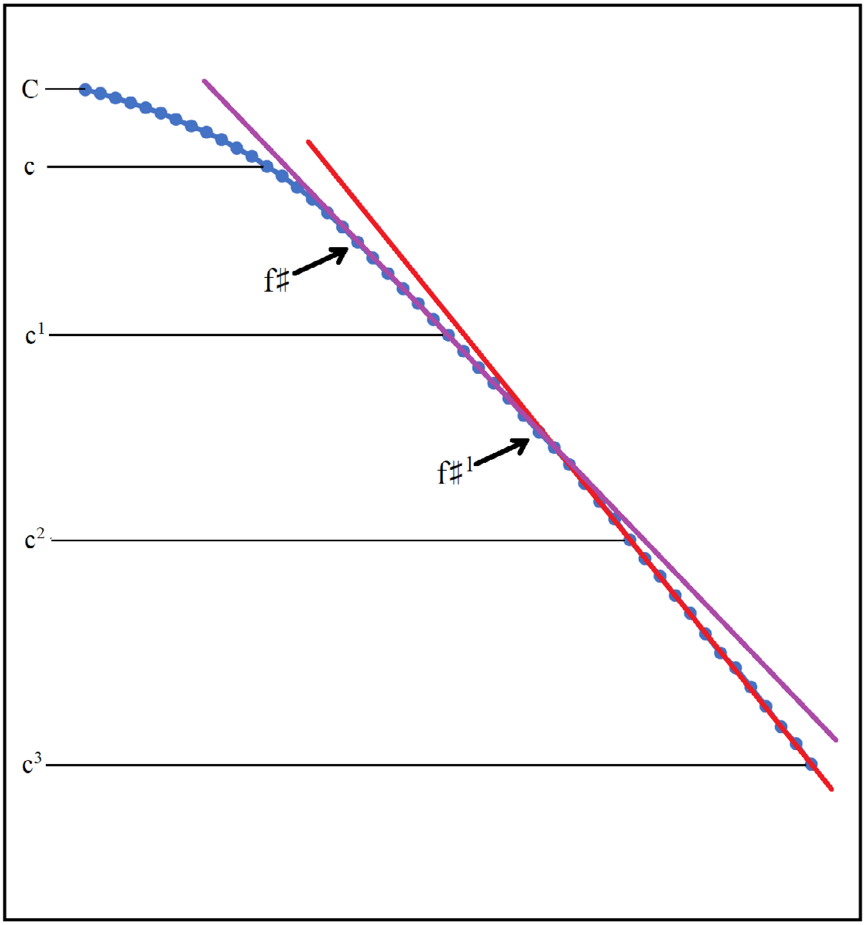

FIGURE 12. String lengths of the MHS piano, graphed with a logarithmic Y-axis. The red and purple straight lines drawn through the data points reveal the different rates of tapering above and below f-sharp[1].

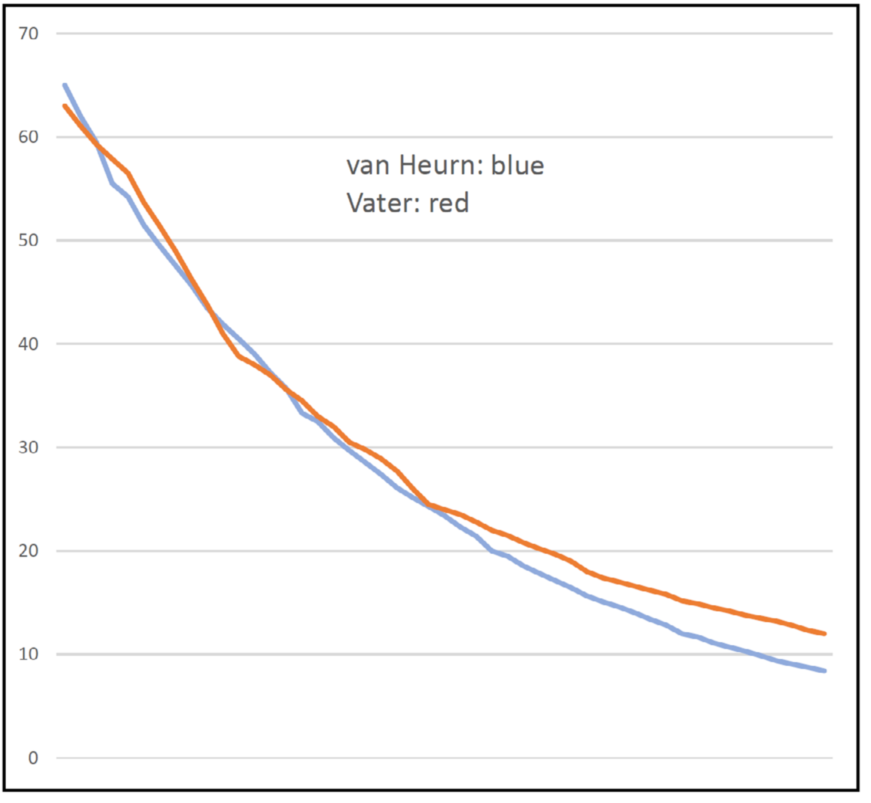

FIGURE 13. Graph of pipe widths from C to c^3 measured from scaling charts by Jan van Heurn (see fig. 4), in blue, and Christian Vater for a *Siefloit* 1⅓′, with its baseline divided haphazardly (*Werkstattbuch*, 22; low C-sharp and E-flat, not included in Vater's chart, have been interpolated). Vater's scaling (in red) results in a randomly erratic curve. Van Heurn's scaling (in blue) results in a more regular curve, in which, however, there is a hiccup around *E-flat* and *E* in each octave and slight wobbling elsewhere, resulting from his division of the baseline according to an approximation of quarter-comma meantone tuning.

Table 4. *C*-string lengths of Saxon and Moravian-American instruments with *C*-oriented compasses, expressed in the makers' presumed unit of measurement.

	in inches (1" = 25.4 mm)					in Saxon *Zoll* (1″ = 23.6 mm)	
	John Clemm(?)	David Tannenberg, 1761	attributed to Tannenberg			Joh. Jacob Donat, Leipzig, 1700	Chr. H. Bohr, Dresden, 1713
	upright piano (MHS)	clavichord (MHS)	clavichord (Schubert Club, St. Paul, Minn., acc. no. A.73.1	clavichord (Smithsonian Institution, Nat'l Museum of American History, cat. no. MI.094886)	clavichord plan	clavichord (Museum für Musikinstrumente, Leipzig)	2-manual spinet (Museum für Musikinstrumente, Leipzig)
							8/9×length of <8'
c³	6 1/4	4 1/4	5 1/16	5 1/16	5	4 10/12	5 10/12
c²	11 1/2	10 3/4	10 3/4	10 7/16	10	10 8/12	10 5/12
c¹	20	18 15/16	19 3/4	19 3/16	20	18 10/12	19 8/12
c	32	31 7/16	31 3/8	32	32 3/4	30 9/12	32
C	39	38 3/8	38 15/16	38 3/16	41 1/2	42 4/12	41 4/12
	Idealized measurements; raw data from John R. Watson	Data from Laurence Libin, "New Insights into Tannenberg's Clavichords," 152.			See fig. 1.	Raw data from Henkel, *Clavichorde*, 38	Raw data from Henkel, *Kielinstrumente*, 41.

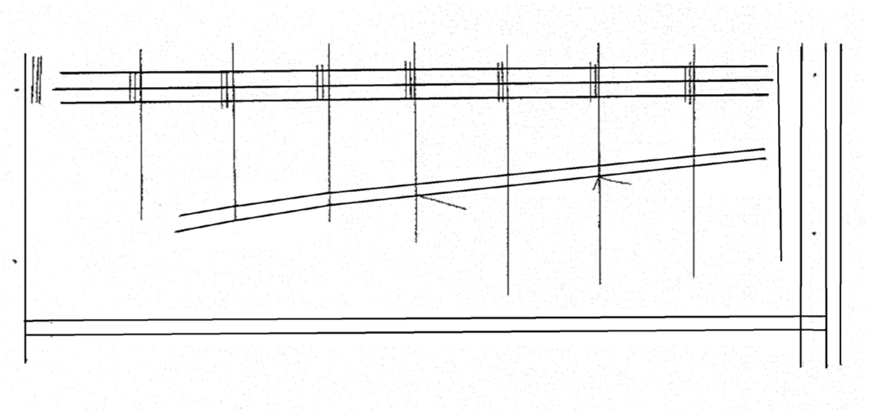

FIGURE 14. Drawing scribed on the bottom board of a harpsichord by Andreas Ruckers, Antwerp, 1636, to lay out a special single-manual design with a chromatic bass instead of the usual C/E short octave. Only the accessible portion under the keyboard is shown, including lines for the nameboard, 8´ nut, registers, and the positions of the jack slots and 8´ strings at each *C* and *F-sharp*. Note that the key notes c^1 and c^2 are marked with additional slashes at the nut. (The instrument, now with two manuals after an eighteenth-century *ravalement*, is in the Cobbe Collection, Hatchlands Park, East Clandon, Surrey.) Drawing by the author.

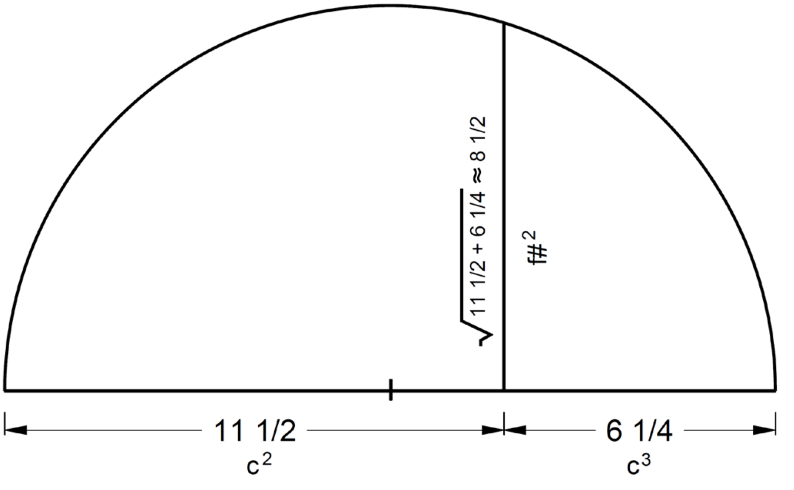

FIGURE 15. The Euclidean method for finding the geometric mean between two lengths, here the length of f-sharp^2 between c^2 of 11½ inches and c^3 of 6¼ inches. Drawing by John Watson.

correspond rather closely with those in clavichords by or attributed to David Tannenberg.[43] Further, evidence of a connection with Saxony could be seen in comparing the Moravian-American instruments with a clavichord made by Johann Jacob Donat in Leipzig, 1700, and with the Bohr spinet. For the latter, however, since we are concerned with the proportions among measurements as much as with the measurements *per se*, we need to adjust Bohr's actual lengths to take into consideration a likely difference in pitch. The relatively short treble strings of the several clavichords are suitable for brass strings tuned to pitches as high as *Chorton*, the high pitch common in Germany, found in at least one of Tannenberg's organs.[44] Bohr's scaling, so very long for the highest 4´ strings, which must have been of iron, is suited to a pitch approximately a whole tone lower, that is, to a *Cammerton* of approximately a^1 = 415 hz. Thus, for purposes of comparison in Table 4, I have multiplied the actual values by 8/9 and converted them to *Zoll*.

From the data in Table 4, we can gather that in the tradition represented by these instruments, the c^1 string in small C-compass instruments (i.e., not harpsichords or grand pianos) at *Chorton* (which might have been regarded loosely as any pitch significantly higher than *Cammerton*) is about 20 inches long, c is about 32, and C, averaging about 39½, is about twice the length of c^1. I am tempted to write that 20, 32, and 40 are the ideal measurements, but it seems better to regard them as "rules of thumb," allowing for some latitude, especially for low C. In the instructions accompanying David Tannenberg's clavichord plan, he wrote that, having marked the lengths of c^3, c^2, and c^1 at 5, 10, and 20 inches, as they are labeled on the drawing, "Farther down there is no reason to measure, since the strings never become too long."[45] In the table, the measurements for c and C, not indicated on the drawing, have been measured and scaled

43. Unfortunately, because, as noted in Watson's article, the spinet made by John Clemm in 1739 (in The Metropolitan Museum of Art, New York), lacks its original bridge and nut, its scaling cannot be measured accurately. In any case, with FF as the lowest note, the foreshortening would be quite different from that in *C*-oriented instruments, as it is in FF-compass spinets by described in Laurence Libin, "Three Spinets from the Workshop of David Tannenberg," forthcoming in *Early Keyboard Journal*.

44. As reported by Philip T. D. Cooper on the website www.davidtannenberg.com, the pitch of the organ, built in 1770, in Zion Lutheran Church, Moselem Springs, Pennsylvania, is 458.2 hz.

45. As transcribed in McGeary, "David Tannenberg and the Clavichord," 103. "Weiter hinunter hat es keine Ursache zu messen, denn die Saiten werden nie zu lang." I have slightly modified his translation.

up from the drawing. Tannenberg seems indeed to have made them a little longer than the normal "rule of thumb" dimensions found in his actual clavichords and in the MHS piano.

The Tapered Scaling of the MHS Piano

Grant O'Brien, in his *Ruckers* book and other publications, has shown the usefulness of graphing stringed-instrument scalings with a logarithmic Y-axis, as we have already done in fig. 11. Sets of string lengths generated by geometric progressions fall along straight lines. The most familiar geometric progression is that of pythagorean scaling, with its 2:1 octave ratio and, for equal temperament, the twelfth root of 2 (1.05946…) as the factor for a semitone. Nineteenth- and early-twentieth-century treatises on piano design,[46] however, recommend tapered scalings generated with geometric progressions based on octave ratios such as 15:8, 15½ : 8, and 17:9, or on octave factors such as the 1.9458608 mentioned by the Viennese makers Wachtl & Bleyer in 1811.[47] Graphed with a logarithmic Y-axis, the string lengths generated by such geometric progressions fall along straight lines with slopes other than that of pythagorean scaling.

One might expect that the MHS piano's scaling would show two straight lines with different slopes, one for the c^1 to c^2 octave, with its ratio of 20 : 11½, and another for the c^2 to c^3 octave, with its ratio of 11½ : 6¼. Fig. 12, a graph of all the string lengths in the instrument, does indeed show two straight-line sections, indicated by the red and purple lines. The two geometrically scaled sections, however, do not conform to the c^1 to c^2 and the c^2 to c^3 octaves, but are from f-sharp to f-sharp^1 (the purple line) and from f-sharp^1 to c^3 (the red line). Strictly, one should say that the inflection points are *approximately* at f-sharp and f-sharp^1, since the differences in the slopes are subtle enough that these points could conceivably be f and f^1 or g and g^1. With *F-sharp*, however, the calculations for generating tapered scalings are relatively simple whether done by arithmetic or by geometry, the latter being more likely for an artisan in the period of the MHS

46. See Siegfried Hansing, *The Pianoforte and its Acoustic Properties* (second ed., Schwerin, 1904); William B. White, *Theory and Practice of Piano Construction* (New York, 1906); and S. Wolfenden, *A Treatise on the Art of Pianoforte Construction* (London, 1916).

47. This is in a notice that Wachtl & Bleyer published in November 1811 in the *Intelligenz-Blatt*, a supplement to the *Allgemeine Musikalische Zeitung*.

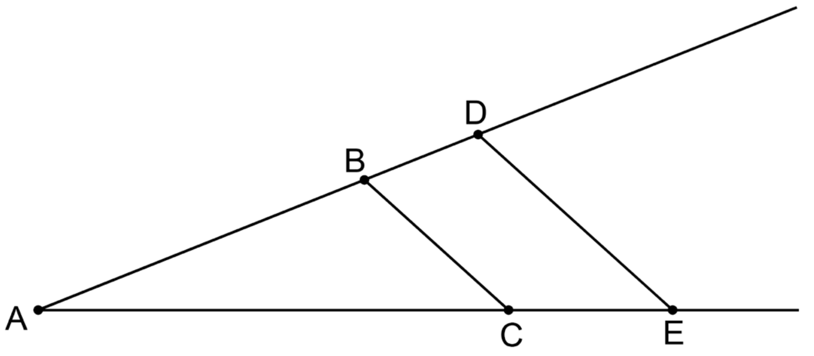

FIGURE 16. Euclidean method of finding the next element in a geometric progression, such that, for example, the length of f-sharp^1 is to c^2 as c^2 is to f-sharp^2. Here the line AB is the length of f-sharp^2; lines AC and AD are the length of c^2. Line DE is parallel to line BC. Line AE is the length of f-sharp^1. Drawing by John Watson.

upright. We will just assume that the inflections are indeed at f-sharp and f-sharp^1.

An organ builder working with a scaling system like those of Class Douwes, Christian Vater, and Jan van Heurn, having determined the plate widths for each *C*, would then have had somehow to determine the widths of all the intervening pipes, *C-sharp* to *B-flat*, so that each width could be transferred with a compass from the scale chart to the sheet of metal, before cutting it to the specific dimensions of the pipe. The division of the baseline of the chart could be done formally, as by van Heurn, or informally, as by Douwes and Vater. In fig. 13 are graphed measurements of pipe widths taken from van Heurn's chart, with its carefully divided baseline, and from Vater's for a *Siefloit* $1\frac{1}{3}'$, with its baseline divided haphazardly. Vater's scaling, represented by the red line, results in a randomly erratic curve. Van Heurn's scaling, represented by the blue line, results in a more regular curve, in which, however, there is a hiccup around *E-flat* and *E* in each octave and slight wobbling elsewhere, resulting from his division of the baseline according to an approximation of quarter-comma meantone tuning. A regular curve would result if the baseline of the scaling chart were divided according to equal temperament,[48] but to do so would involve the use of logarithms or other mathematical techniques beyond the ken of instrument makers until the late eighteenth century

48. As in the altered version of van Heurn's chart in Mahrenholz, *Die Berechnung der Orgelpfeifen-Mensuren*, 47.

(see Excursus 1, below). Fortunately, for organ builders it hardly matters, whatever temperament is chosen for the final tuning, if a pipe is slightly wider or narrower than it should be in theory.

Makers of stringed-keyboard instruments need to determine the position and curve of the bridge, but any curve generated from traditional pipe-scaling methods would, if followed exactly with all the wobbles seen in fig. 13, be unsuitable for the bridge of a stringed-keyboard instrument.[49] Fortunately, makers of harpsichords, clavichords, and early pianos did not need to measure the lengths of every string to set their bridges, nor did the length of each string need to conform exactly to the theoretical length prescribed for any particular temperament. Setting only one or, better, two points in each octave is sufficient. On Ruckers instruments, for example, there are positioning pin marks along the bridge at the positions for *C* and *F-sharp* in each octave.[50] When members of the Ruckers family designed special models by scribing plans on the bottom boards, they marked just the *C* and *F-sharp* strings, as seen, for example, in a harpsichord by Andreas Ruckers, 1636, which originally was a single-manual harpsichord with a chromatic bass rather than the usual short octave (fig. 14).[51] With marks or pins at the position of each *C* and *F-sharp*, the curve of the bridge could be drawn with a spline to guide the stylus.[52] Some makers might have been

49. The problem can be seen in a theoretical harpsichord designed by Salomon de Caus with a strictly pythagorean scaling for an unequal tuning, as described and illustrated in his *Institution harmonique* (Frankfurt am Main, 1615), fol. 18–18v. The strings pass over individual bridges, which, if they were combined into a single bridge would form an irregular curve.

50. See O'Brien, *Ruckers*, 106–07. Instead of *F-sharp* to supplement the key-note *C* positions, some makers used *G*, as in an anonymous Flemish harpsichord described in Grant O'Brien, "An Analysis of the Origins of a Large Franco-Flemish Harpsichord—Would a Ruckers by Any Other Name Sound as Sweet?" *Early Keyboard Journal* 22 (2004): 49–80, specifically 62. Other makers might have used *F*.

51. I will describe the several such Ruckers drawings in a future article.

52. There is little, if any, documentary or iconographical evidence for the use of splines in the period. The *Oxford English Dictionary*'s earliest instance of "spline" in this sense is from 1891. There was some literary knowledge of "Lesbian rules," which Aristotle described as a "leaden rule used by Lesbian builders" (i.e., builders on the island of Lesbos), which "is not rigid but can be bent to the shape of the stone": see https://en.wikipedia.org/wiki/Lesbian_rule.

An old engraving showing two advanced forms of adjustable spline, said to be from 1690, is reproduced in the introduction of Brian Lavary's edition of *Deane's Doctrine of Naval Architecture, 1670* (London: Conway Maritime Press, 1981), 20. In one of these tools, the curve is adjusted by three screws. In the other, resembling the bow of a stringed instrument, the curve of the stick is changed by adjusting the tension of a cord connecting its ends. Since the stick is tapered in thickness, the radius of the curve varies along its length. (A modern version of this tool, called an "asymmetric drawing bow," can be seen more casual in drawing their bridge curves. In the instructions accompanying David Tannenberg's clavichord drawing, he wrote that, having marked the positions of c^3, c^2, and c^1, "one, measuring by eye, makes the curve from one *C* to the other *C*."[53] Obviously, however, more care must have been taken in marking out the refined and subtle tapered scaling of the MHS piano.

The maker of the MHS piano, having chosen the lengths of the *C* strings and having decided to make inflection points for changes in the rate of tapering in the middle of the octaves c to c^1 and c^1 to c^2, would then have calculated the lengths for a note around the middle of these octaves and, for accurate placement of the bridge, the top and bottom octaves as well. For *F-sharp*, which is exactly in the middle of the octave, six notes above and below *C*, the calculations are especially simple. When the Ruckers determined their treble *F-sharp* lengths according to pythagorean scaling, they had, as Herbert Heyde has pointed out,[54] merely to construct a square with sides of 7 *duimen*, the length of c^3, the diagonal of which would be the length of f-sharp^2, the geometric mean between seven and fourteen. This is essentially the same method shown by Athanasius Kircher and Salomon de Caus for pipe scaling with the octave factor of 1 to the square root of 2 (as in figs. 5 and 6).

The procedure for the HMS upright, with its tapered scaling, would have been somewhat more complicated, but well within the bounds of elementary arithmetic or Euclidean geometry. Fig. 15 shows the method, well known from Euclid's *Elements* (Book 6, Proposition 13), for finding the geometric mean between the 11½ inches of c^2 and the 6¼ of c^3. One draws a line 11½ plus 6¼ inches long, finds the midpoint and, with the compass centered there, draws a semicircle from end to end of the line.

at https://www.leevalley.com/en-us/shop/tools/hand-tools/marking-and-measuring/marking-accessories/44631-lee-valley-drawing-bows.) I have not been able to find the source of the engraving in books published around 1690.

One should note that the *Doctrine* by Sir Anthony Deane (1633–1721) describes curves drawn only with compasses. It would seem that, as described in numerous historical sources, complex curves were generally drawn by joining circular arcs of different radii (as discussed in John Koster, "Traditional Iberian Harpsichord Making in Its European Context," *Galpin Society Journal* (2008): 3–78, especially 32–37). Nevertheless, it seems necessary in most cases to assume that harpsichord and early piano makers used splines to form the curves of their bridges.

53. As transcribed in McGeary, "David Tannenberg and the Clavichord," 103. "Alsdann macht man die Biegung von einem C zum anderen C nach dem Augenmass."

54. Herbert Heyde, *Musikinstrumentenbau, 15.–19. Jahrhundert*, 164.

A perpendicular line is drawn from the point on the baseline 11½ inches from one end and 6¼ from the other. The length of this line to the arc is the geometric mean. For the MHS piano, this mean, the length of f-sharp^2, could alternatively have been calculated numerically by multiplying 11½ and 6¼ and extracting the square root of the product, resulting in 8.4779…, almost exactly the instrument's actual f-sharp^2 length of 215 mm or 8.46 inches.

To find the length of f-sharp^1, the lowest note of the portion of the MHS piano scaling corresponding to the red line in fig. 12, the geometric progression can be continued by the construction in fig. 16, also based on Euclid (Book 6, Prop. 2). From point A, two lines are drawn at an arbitrary acute angle. AB is the length of f-sharp^2, and AC and AD are the length of c^2. A line is drawn from B to C and, parallel to it, a line from D to E. AE, then, is the length of f-sharp^1. The length of f-sharp^1 could alternatively be calculated numerically by the Rule of Three (see Excursus 2), by which, in this instance, one would multiply the c^2 length of 11½ by itself and divide the product, 132¼, by the length of f-sharp^2, giving 15.5993…. If, for convenience, the numbers were rounded to 132 and 8½, the result would be 15.5294…, which itself could be rounded to 15½. (Such rounding would be allowed by Bendeler, who, coming to 1037-$^{3}/_{81}$ in a calculation, rounded it to 1037, because "one need not pay attention to the fraction, since it does not even amount to the thickness of a hair."[55]) All these results are close to the actual f-sharp1 length of 392 mm or 15.43 inches.

If the maker of the MHS upright had continued the geometric progression of the top octave and a half down to c^1, this would have been 21.16 inches long. He decided, however, to taper the octave below f-sharp^1 somewhat more steeply, corresponding to the purple line in fig. 12. Choosing 20 inches as the length of c^1, he could find the length of f-sharp from this and f-sharp^1 by the methods already used for finding f-sharp^1 from c^2 and f-sharp^2. With the 20-inch c^1 and 15½-inch f-sharp^1, the numerical method results in an f-sharp of 25.8 inches or 655.5 mm, almost exactly the actual length of 657 mm. It seems plausible that at this stage of the scaling process, and perhaps the previous, the maker of the MHS upright would have used the numerical Rule of Three method rather than constructing large geometrical diagrams. For the initial calculation of the length of

55. Bendeler, *Organopoeia*, 12. "der Bruch ist nicht zu attendieren, weil er noch nicht ein Haar breit ausmacht."

f-sharp², however, the geometrical method of finding the mean between 11½ and 6¼ would have been preferable, as quicker and easier than the numerical method, which requires the extraction of a square root.

The tapered scaling ends at f-sharp, at which the foreshortening of the lowest octave and a half begins, as does the reverse curve of the bridge. With the lengths of the two lowest *C* strings set by rule of thumb at 32 and 39 inches, that leaves F-sharp. This string, 917.5 mm or 36.12 inches long, might have been set arbitrarily, by eye or rule of thumb, at 36 inches. Alternatively, however, C might have been set virtually at the "ideal" 40 inches (twice the length of c¹), from which one inch was subtracted in the manner of what has been called the "bass-hook adjustment."[56] The 36 inches for F-sharp, then, would just be the arithmetic mean (average), halfway between the 32 and (virtual) 40-inch lengths of the c and C strings.

As noted above, tapered scaling for stringed-keyboard instruments seems to have been a relatively new concept in Central-German harpsichord making when Christoph Heinrich Bohr made his two-manual spinet in 1713. This scaling technique and that of varying the rate of taper for bent scales, as in the MHS piano, was quite likely derived from organ-pipe scaling techniques. A direct or indirect connection between John Clemm and Dresden organ builders who made harpsichords with tapered scaling is plausible.

Excursus 1

The Use of Logarithms in Keyboard Instrument Making

Logarithms, developed by John Napier (1550–1517) and explained in his book *Mirifici Logarithmorum Canonis Descriptio* (Edinburgh, 1614), reduce the calculation of roots of any degree to a simple matter of division. In the days before electronic calculators, to find, for example, the twelfth root of two, which is the factor of an equal-tempered semitone, one merely had to look up the logarithm of 2 in a table and divide this by twelve. The quotient, which is the logarithm of the desired twelfth root, is then looked up in the table to find the corresponding number, which is the answer

56. John Koster, "Three Early Transposing Two-Manual Harpsichords of the Antwerp School," *Galpin Society Journal* 57 (2004): 81–116, especially 94–98.

to the problem. (This is not to deny that some experience and skill are needed to use tables of logarithms, particularly to estimate answers that fall somewhere between the listed values.) Or if, for example, one were to choose string length X for f^1 and length Y for c^3 and wished to determine the factor by which each semitone would be larger than the next, one would divde X by Y and find the nineteenth root of the quotient. This is easily done with logarithms but impossible with plain arithmetic or by Euclidean geometry.

Tables of logarithms suitable for practical use, printed soon after Napier's initial publication, enabled scientists and mathematicians of a musical bent, such as Lemme Rossi (circa 1602–1673) and Christian Huyghens (1629–1695) to use them for abstruse studies of temperaments.[57] Perhaps the earliest source to espouse logarithms, in a way that could plausibly have been put to practical use by instrument makers, was Georg Andreas Sorge's *Ausführliche und deutliche Anweisung zur Rational-Rechnung und der damit verknüpfften Ausmessung and Abtheilung des Monochords* (Lobenstein, 1749) which, while presenting the mathematics of logarithms in general, was mainly directed towards their use in calculating temperaments. Jacob Adlung's *Anleitung zu der musikalischen Gelahrtheit* (Erfurt, 1758) cites the book and mentions rather vaguely that "from this book one can learn the hitherto existing and the subsequent methods of calculation."[58] I have, however, found no evidence that any instrument maker made practical use of Sorge's instructions and table of logarithms. It is clear that David Tannenberg and other Pennsylvania-German organ builders active after John Clemm's death were familiar with Sorge's later work, *Die geheim gehaltene Kunst der Mensuration der Orgelpfeiffen*, which the author began to offer for sale in manuscript copies about 1760 (one of reached the Moravian-Americans in 1764), and with his printed *Der in der Rechen- und Meßkunst wohlerfahrne Orgelbaumeister* (Lobenstein, 1773).[59] These treatises describe the use of logarithms for calculating geometric progressions for organ-pipe scaling in which the width halves or doubles

57. See J. Murray Barbour, *Tuning and Temperament: A Historical Survey, second ed.* (East Lansing: Michigan State University Press, 1953), 30 and 118.

58. Jacob Adlung, *Anleitung zu der musikalischen Gelahrtheit* (Erfurt, 1758), 282. "Aus diesem Buche kann man die bisherigen und folgenden Rechnungsarten lernen."

59. See Carl O. Bleyle's commentary in his facsimile edition and translation of Sorge, *The Secretly Kept Art of the Scaling of Organ Pipes* (Buren: Frits Knuf, 1978), and Raymond J. Brunner, *"That Ingenious Business": Pennsylvania German Organ Builders* (Birdsboro, Penn.: The Pennsylvania German Society, 1990), chapter 4.

at the 14th, 15th, or 16th pipe above or below. The American builders, however, evidently worked with measurements taken from Sorge's charts, without calculating scales themselves.

Although logarithms continued to be put to musical use in treatises about temperament, such as Daniel Gottlob Türk's *Anleitung zu Temperaturberechung* (Halle, 1808), Sorge's innovative application of logarithmic methods in practical instrument making evidently fell by the wayside. The first known instrument makers likely to have used logarithms themselves seem to have been the Viennese piano makers Wachtl & Bleyer. According to their notice published in November 1811 in the *Intelligenz-Blatt*, a supplement to the *Allgemeine Musikalische Zeitung*, they used a monochord to determine the ideal length, thickness, and tension of the f and f^4 strings, and from these they calculated the dimensions of the intervening notes. The only detail they provide is that this resulted in an octave factor of 1.9458608, but they must have used logarithms to determine the 48th root of the quotient of the two known string lengths. (This quotient would have been the octave factor raised to the fourth power, i.e., 14.33663.) The process of using logarithms for such a calculation of a tapered string scaling was later described explicitly in Carl Kützing (1798–1862), *Das Wissenschaftliche der Fortepiano-Baukunst* (Bern, Chur, and Leipzig, 1844), who found that the logarithm of the 48th root of the quotient of the f and f^4 lengths, which for him were respectively 860 and 60 mm, was 0.02409. With the calculator in my mobile phone, I find that this root, the factor for a semitone, is 1.057036…, corresponding to an octave factor of 1.9457184…, which is vanishingly close to Wachtl & Bleyer's. Kützing must somehow have known or figured out what lengths Wachtl & Bleyer had used for f and f^4. The slight difference between his and their octave ratios was probably due to the use of different units of measurement and the effect of rounding them off to the closest fraction of a *Zoll* or millimeter.

Eleven years before his treatise of 1844, Kützing's *Theoretisch-praktisches Handbuch der Fortepiano-Baukunst* (Bern and Chur, 1833) provided (pp. 18–21 and plate 1) instructions along traditional lines for using a two-foot-long sector (or proportional compass) marked with an equal-tempered scale to determine string lengths for pythagorean scaling, that is, with octave ratio 2:1. The widespread adoption of logarithmic methods for calculating geometric progressions for both organ-pipe and piano-string scalings, at least in German-speaking regions, can be seen during the following decade in publications by Kützing and by Johann Gottlob Töpfer (1791–1870):

- Töpfer's *Die Orgelbau-Kunst nach einer neuen Theorie dargestellt und auf mathematische und physikalische Grundsätze gestützt* (Weimar, 1833) and *Erster Nachtrag zur Orgelbau-Kunst welcher die Vervollständigung der Mensuren zu den Labialstimmen u[nd] die Theorie der Zungenstimmen mit den dazu gehörigen Mensur-Tabellen derselben* (Weimar, 1834) describe in great detail the use of logarithms to calculate geometric progressions for organ-pipe widths. These were based on octave factors for the cross-sectional areas of the pipes, most notably the square root of eight, equivalent to a width factor of the fourth root of eight, approximately 1.68179, which eventually was accepted as *Normalmensur* in nineteenth- and twentieth-century organ building. Within sixteen years, Töpfer's method of scaling was included in the third volume of Marie-Pierre Hamel's *Nouveau manuel complet du facteur d'orgues* (Paris, 1849).

- Kützing's *Theoretisch-praktisches Handbuch der Orgelbaukunst* (Bern, Chur, and Leipzig, 1836; *R* 1843) acknowledges the worth of Töpfer's work but regards it as insufficiently practical. Ideal diameters for Principal pipes for CC (16´) and c^5 ($\frac{1}{8}$´) are chosen and, using logarithms, the diameters of all the intervening pipes are calculated as a geometric progression with an octave factor of the eighth root of 54, approximately 1.64645. In his foreword (p. iv) Kützing remarks "Since I know that the majority of organ builders are completely unfamiliar with the elements of mathematics, I have tried to do everything for this class [of people] to become as capable as possible, especially since it is precisely they who most need such support."[60]

- Kützing's *Beiträge zur praktischen Akustik als Nachtrag zur Fortepiano- und Orgelbaukunst* (Bern, Chur, and Leipzig, 1838) mentions the use of logarithms in calculating the frequencies of all the notes from CCC (32´) to c^7 (1/32´). The table of piano string lengths from f to g^4 is strictly pythagorean (octave ratio 2:1, with the semitone factor of the twelfth root of two).

- Töpfer, *Abhandlung über den Saitenbezug der Pianoforte's [*sic*] in Flügel- und Tafel-Form* (Leipzig, 1842), taking into consideration the diameters and tensions of the strings, makes elaborate calculations, in which the three-octave factor is determined to be four times the eighth root of 128 (equivalent to an octave factor of 1.94306…). The geometric progression

60. Carl Kützing, *Theoretisch-praktisches Handbuch der Orgelbaukunst* (Bern, Chur, and Leipzig, 1836; *R* 1843), iv. "Da ich weiß, daß de Mehrzahl der Orgelbauer mit den Elementen der Mathematik ganz unbekannt ist, so habe ich mich bemüht alles aufzusuchen, um auch dieser Klasse so nützlich als möglich, zu werden, zumal da gerade sie dergleichen Unterstützungen am meisten bedarf."

for determining the dimensions of all the strings is found "by the method known to every mathematician."[61] For an alternative scheme, with thicker strings for the lower notes, the factor of two-and-a-half octaves is half the square root of 32 times the eighth root of 128 (equivalent to an octave factor of 1.93186…). The results of these calculations for tapered scalings and of others for the foreshortened bass strings are presented in tables.

- Töpfer, *Die Orgel, Zweck und Beschaffenheit ihrer Theile, Gesetze ihrer Construction, and Wahl der dazu gehörigen Materialen . . . Ein Handbuch . . .* (Erfurt, 1843), without going into the detailed mathematics of logarithms, advocates the cross-sectional octave factor of the square root of eight, developed in his earlier publications.

- Kützing, *Das Wissenschaftliche der Fortepiano-Baukunst* (Bern, Chur, and Leipzig, 1844), as mentioned above, presents a tapered scaling practically identical to that advertised by Wachtl & Bleyer in 1811. A notable innovation, however, was Kützing's use of the metric system, expressing lengths in millimeters, weights in grams, and tensions in kilograms, rather than the traditional units (*Zoll*, etc.) used in his and Töpfer's earlier publications. Kützing's tables give string lengths for each note, calculated to a tenth of a millimeter, and even to a hundredth in the top octave.

In aggregate, these works of the 1830s and '40s by Töpfer and Kützing stand in the long tradition, particularly strong in the Germanic countries, in which the building of organs and of stringed keyboards were closely related activities. This was the tradition within which Johann Philipp Bendeler had worked a hundred and fifty years earlier, as evident in the full title of his book, which is, in translation:

> Organopoeia, or Instruction on how to build an organ according to its main parts, such as scaling, marking out its windchests, problems of wind supply, tuning and temperament etc., on true mathematical grounds; with a supplement on how to convert all bad-sounding spinets, harpsichords, etc. to have a lovely tone without changing the soundboard; likewise how to quill them well.[62]

61. Johann Gottlob Töpfer, *Abhandlung über den Saitenbezug der Pianoforte's in Flügel- und Tafel-Form* (Leipzig, 1842), 25. "auf die jedem Mathematiker bekannte Art."

62. Bendeler, *Organopoeia, Oder: Unterweisung, Wie eine Orgel nach ihren Hauptstücken, als Mensuriren, Abtheilung derer Laden, Zufall des Windes, Stimmung oder Temperatur etc., aus wahren Mathematischen Gründen zuerbauen: Sammt einer Zugabe, Wie alle übel-klingende Spinette, Clavicimbel, etc. zu einem lieblichen Klange, ohne Veränderung der Decke, zu bringen; Ingleichen, wie sie wohl zubekielen.*

Although Töpfer and Kützing's publications represented the final flowering of that tradition, they can be regarded as transitional, providing new approaches for the design of both organs and pianos.

Excursus 2

The Rule of Three

The Rule of Three, sometimes called the Rule of Proportions or the Golden Rule, is a historical method for solving problems such as that posed as an example by Robert Record in his primer of arithmetic, *The Grounde of Arts* (London, 1542): "If you paye for your borde for thre[e] monthes 16 s[hillings], how much shall you pay for 8 monthes?" We would set up an equation of the form:

$a \div b = c \div x$
or here
$3 \div 8 = 16 \div x.$

Thus:
$X = bc \div a$
or here
$x = (8 \text{ times } 16) \div 3 = 128 \div 3 = 42\tfrac{2}{3}.$

The Rule of Three was enormously useful in trade and commerce, to figure costs, to convert one currency to another or one local unit of measurement to another, to figure quantities in recipes, and so on. It was constantly put to use in Bendeler's *Organopoeia*. For calculating the length of f-sharp^1 in the MHS piano, the numbers b and c in the equation are both the length of c^2, 11½ inches, and a is the length of f-sharp^2, which we might round to 8½. Thus:
the length of f-sharp^1 $= (11\tfrac{1}{2} \text{ times } 11\tfrac{1}{2}) \div 8\tfrac{1}{2} = 132\tfrac{1}{4} \div 8\tfrac{1}{2} = 15\tfrac{19}{34}.$

Or if we rounded further:
$$132 \div 8\tfrac{1}{2} = 263 \div 17 = 15\tfrac{8}{17} \approx 15\tfrac{8}{16} = 15\tfrac{1}{2}.$$